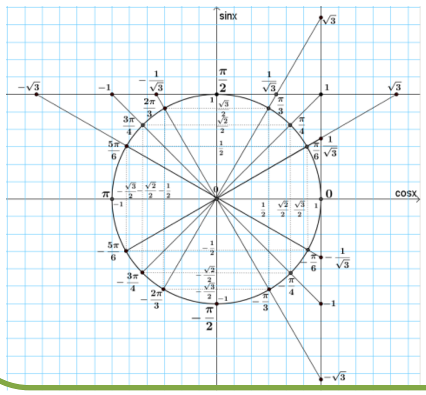
Тема: Преобразования тригонометрических выражений

Профиль



- Алгебра 10 / Преобразование тригонометрических выражений/ Банк заданий с решениями

Преобразование тригонометрических выражений
Банк заданий с решениями

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I. Вычислите

№1. $\sin 540^\circ$

№2. $\sin 210^\circ$

№3. $\cos 330^\circ$

№4. $\cos\left(-855^\circ\right)$

№5. $tg300^\circ$

№6. $\cos 240^\circ$

№7. $tg3810^\circ$

№8. $ctg2,5\pi$

№9. $\cos 2\frac{2}{3}\pi$

№10. $\sin 7\frac{5}{6}\pi$

№11. $\cos 105^\circ$

№12. $\sin 285^\circ$

№13. $\sin 20^\circ \cdot \cos 70^\circ + \sin^2 110^\circ \cdot \cos^2 250^\circ + \sin^2 290^\circ \cdot \cos^2 340^\circ$

№14. $\left(\sin 75^\circ + \sin 100^\circ\right)\cdot\left(\sin 260^\circ - \sin 285^\circ\right) + \left(\sin 165^\circ + \sin 190^\circ\right)\cdot\left(\cos 75^\circ - \cos 100^\circ\right)$

№15. $\left(\frac{tg^2 590^\circ}{\cos^2 320^\circ} + \frac{\sin 111^\circ}{\cos 159^\circ}\right)\cdot\left(\frac{\cos 279^\circ}{\sin 549^\circ} + \frac{ctg^2 950^\circ}{\sin^2 400^\circ}\right)$

№16. $\sin 167^\circ \cdot \sin 107^\circ + \sin 257^\circ \cdot \sin 197^\circ$

№17. $\sin\frac{7\pi}{4} + \cos\frac{17\pi}{4} + tg\frac{19\pi}{4} + ctg\frac{7\pi}{4}$

№18. $\sin\left(-\frac{7\pi}{4}\right) + \cos\frac{7\pi}{4} + tg\frac{15\pi}{4} - ctg\left(-\frac{7\pi}{4}\right)$

№19. $\frac{1}{1-2\cos 30^\circ} + \frac{1}{1+2\sin 60^\circ}$

№20. $\frac{1}{tg60^\circ - 1} - \frac{1}{ctg30^\circ + 1}$

№21. $\frac{tg\frac{13\pi}{4}}{\cos\frac{7\pi}{4} + 1}$

№22. $\frac{ctg\left(-\frac{7\pi}{4}\right)}{\sin\frac{13\pi}{4} + 1}$

№23. $\frac{1}{\cos 1110^\circ + \cos 2220^\circ + \cos 3330^\circ}$

№24. $\left(\cos 1140^\circ + \sin 2280^\circ + \sin 3420^\circ\right)^{-1}$

№25. $tg615^\circ + tg375^\circ$

№26. $ctg\frac{13\pi}{12} - ctg\frac{5\pi}{12}$

№27. $\left(\sin\frac{7\pi}{18} - \sin\frac{\pi}{9}\right):\left(\cos\frac{7\pi}{18} - \cos\frac{\pi}{9}\right)$

№28. $\frac{6\sin 35^\circ \cdot \sin 55^\circ}{\cos 20^\circ}$

№29. $\frac{\sin 54^\circ}{\cos 63^\circ \cdot \sin 117^\circ}$

№30. $2\sin^2\frac{\pi}{12} - 1$

№31. $\cos 195^\circ \cdot \cos 105^\circ + \sin 105^\circ \cdot \cos 75^\circ$

№32. $\sin 23^\circ \cdot \sin 53^\circ + \sin 67^\circ \cdot \sin 37^\circ$

№33. $ctg5^\circ \cdot ctg10^\circ \cdot ctg15^\circ \cdot \ldots \cdot ctg85^\circ$

№34. $tg3^\circ \cdot tg6^\circ \cdot tg9^\circ \cdot \ldots \cdot tg87^\circ$

№35. $\cos 20^\circ + \cos 40^\circ + \cos 60^\circ + \ldots + \cos 160^\circ + \cos 180^\circ$

№36. $tg20^\circ + tg40^\circ + tg60^\circ + \ldots + tg160^\circ + tg180^\circ$

№37. $\sin 0^\circ + \sin 1^\circ + \sin 2^\circ + \ldots + \sin 359^\circ + \sin 360^\circ$

№38. $ctg15^\circ + ctg30^\circ + ctg45^\circ + \ldots + ctg165^\circ$

№39. $\sin^4 \frac{7\pi}{8} + \cos^4 \frac{\pi}{8}$

№40. $\left(\sin \frac{5\pi}{8} + \cos \frac{3\pi}{8}\right)^2$

№41. $\sin^6 \frac{\pi}{8} + \cos^6 \frac{7\pi}{8}$

№42. $\sin^6 \frac{\pi}{8} - \cos^6 \frac{7\pi}{8}$

№43. $\frac{\sin 70^\circ + \cos 40^\circ}{\sin 280^\circ}$

№44. $\frac{\cos 20^\circ - \sin 50^\circ}{\cos 280^\circ}$

№45. $\frac{\sin 50^\circ + 2\sin 10^\circ}{\cos 50^\circ}$

№46. $\frac{\cos 35^\circ + 2\cos 85^\circ}{\sqrt{3}\cos 55^\circ}$

№47. $\frac{\sin 50^\circ \cdot \cos 12^\circ - \sin 40^\circ \cdot \cos 78^\circ}{\cos 68^\circ - \sqrt{3}\sin 68^\circ}$

№48. $\frac{2\sin^2 70^\circ - 1}{2ctg115^\circ \cdot \cos^2 155^\circ}$

№49. $8\left(\sqrt{3} - \sqrt{2}\right)\sin 52^\circ 30' \cdot \cos 7^\circ 30'$

№50. $\sin^2 \frac{\pi}{13} + \sin^2 \frac{11\pi}{26}$

№51. $\frac{\sqrt{2\left(1 - \sin 82^\circ\right)}}{\sin 4^\circ}$

№52. $\frac{\sqrt{3} + tg\frac{11\pi}{12}}{1 + \sqrt{3}tg\frac{\pi}{12}}$

№53. $\frac{\frac{1}{\sqrt{3}} + tg\frac{13\pi}{12}}{\sqrt{3} - tg\frac{\pi}{12}}$

№54. $\left(\left(tg^2\frac{7\pi}{24} - tg^2\frac{\pi}{24}\right) : \left(1 - tg^2\frac{7\pi}{24} \cdot tg^2\frac{\pi}{24}\right)\right)^2$

№55. $\sqrt{3}\left(tg\frac{51\pi}{36} - tg\frac{13\pi}{12}\right)$

№56. $\frac{\cos 2{,}9\pi \cdot tg2{,}4\pi \cdot tg1{,}1\pi}{\cos 0{,}9\pi}$

№57. $\frac{\cos 0{,}9\pi \cdot ctg0{,}4\pi \cdot ctg2{,}1\pi}{\cos 2{,}1\pi}$

II. Упростите выражения

№58. $\sin\left(\alpha-\frac{\pi}{2}\right)\cdot\cos\left(\alpha-\frac{3\pi}{2}\right)\cdot\cos(\alpha-\pi)$

№59. $\cos\left(\alpha-\frac{\pi}{2}\right)\cdot tg\left(\alpha-\frac{3\pi}{2}\right)\cdot tg(8\pi-\alpha)$

№60. $\dfrac{\sin\frac{3\pi+2\alpha}{2}\cdot tg\frac{2\alpha-\pi}{2}}{\cos(\pi+\alpha)}$

№61. $\dfrac{\cos\frac{5\pi-2\alpha}{2}\cdot ctg\frac{2\alpha-3\pi}{2}}{\sin(\alpha-3\pi)}$

№62. $\sin\left(\frac{3\pi}{2}-\alpha\right)tg(\alpha-\pi)-\cos\left(\frac{15\pi}{2}-\alpha\right)$

№63. $\cos\left(\frac{19\pi}{2}-\alpha\right)+\sin\left(\alpha-\frac{3\pi}{2}\right)tg(\pi-\alpha)$

№64. $\dfrac{\cos(29,5\pi+2)\cdot ctg(19,5\pi-1)}{\sin\left(\sqrt{1-4\pi+4\pi^2}\right)\cos\left(\sqrt{16\pi^2+8\pi+1}\right)}$

№65. $\dfrac{\sin(2-17,5\pi)\cdot tg(9,5\pi-1)}{\sin\left(\sqrt{4-4\pi+\pi^2}\right)\cos\left(\sqrt{4\pi^2+8\pi+4}\right)}$

№66. $tg100^\circ+\dfrac{\sin 530^\circ}{1+\sin 640^\circ}$

№67. $ctg0,4\pi-\dfrac{\cos 1,1\pi}{1-\cos 0,6\pi}$

№68. $tg(360^\circ-x)+ctg(270^\circ-x)+tg(180^\circ-x)+ctg(90^\circ-x)$

№69. $\sin\left(\frac{\pi}{2}+x\right)+\cos(\pi+x)+tg\left(\frac{3\pi}{2}-x\right)+ctg(2\pi-x)$

№70. $\sin x-\sin(x-90^\circ)-\sin(x-180^\circ)-\sin(x-270^\circ)-\sin(x-360^\circ)$

№71. $\cos(x+45^\circ)+\cos(x+135^\circ)+\cos(x+225^\circ)+\cos(x+315^\circ)$

№72. $tg(45^\circ-x)\cdot tg(45^\circ+x)$

№73. $ctg\left(\frac{\pi}{4}+x\right)\cdot ctg\left(\frac{\pi}{4}-x\right)$

№74. $\sin\left(90^\circ+x\right)\cdot\sin\left(180^\circ-x\right)\cdot\left(tg\left(180^\circ+x\right)+tg\left(270^\circ-x\right)\right)$

№75. $\sin\left(x-\frac{\pi}{2}\right)\cdot\sin\left(x+\frac{\pi}{2}\right)-\sin^2\left(x-\pi\right)\cdot\sin^2\left(x+\pi\right)-\cos^2\left(x+\pi\right)\cdot\cos^2\left(x-\frac{3\pi}{2}\right)$

№76. $1-\sin\left(x-2\pi\right)\cdot\cos\left(x-\frac{3\pi}{2}\right)-tg\left(\pi-x\right)\cdot tg\left(\frac{3\pi}{2}-x\right)-2\cos^2\left(\pi+x\right)$

№77. $\sin^2\left(\pi-x\right)+tg^2\left(\pi-x\right)\cdot tg^2\left(\frac{3\pi}{2}+x\right)+\sin\left(\frac{\pi}{2}+x\right)\cdot\cos\left(x-2\pi\right)$

№78. $\dfrac{ctg\left(\alpha-\frac{\pi}{2}\right)\left(\sin\left(\alpha-\frac{3\pi}{2}\right)-\sin\left(\pi+\alpha\right)\right)}{tg\left(\pi+\alpha\right)\left(\cos\left(\alpha+2\pi\right)+\sin\left(\alpha-2\pi\right)\right)}$

№79. $\dfrac{tg\left(\frac{3\pi}{2}-x\right)\cdot\cos\left(x-\frac{7\pi}{2}\right)}{\cos\left(10\pi-x\right)}+\cos\left(x-5\pi\right)\sin\left(5\pi-x\right)+\cos\left(5\pi+x\right)\sin\left(x-\frac{9\pi}{2}\right)$

№80. $\left(ctg\left(6,5\pi-\alpha\right)\cdot\cos\left(-\alpha\right)+\cos\left(\pi-\alpha\right)\right)^2+\dfrac{2\sin^2\left(\pi-\alpha\right)}{tg\left(\alpha-\pi\right)}$

№81. $\left(\sin\left(\frac{\pi}{2}+x\right)+\sin\left(\pi-x\right)\right)^2+\left(\cos\left(1,5\pi-x\right)+\cos\left(2\pi-x\right)\right)^2$

№82. $\left(\dfrac{\cos\left(2,5\pi+\alpha\right)}{ctg\left(3\pi-\alpha\right)}-\sin\left(-\alpha\right)\cdot tg\left(\frac{5\pi}{2}-\alpha\right)\right)^2+\dfrac{tg\alpha}{tg\left(\frac{3\pi}{2}+\alpha\right)}$

№83. $\dfrac{tg\left(\frac{3\pi}{2}-\alpha\right)-\cos\left(\pi-\alpha\right)\sin\left(3\pi+\alpha\right)}{\left(\cos\left(3,5\pi+\alpha\right)+\sin\left(1,5\pi+\alpha\right)\right)^2-1}$

№84. $\dfrac{\sin 2\alpha}{\sin^2\left(\frac{\pi}{2}+\alpha\right)-\sin^2\left(\pi+\alpha\right)}$ и найти его числовое значение при $\alpha=\pi/8$.

№85. $\sqrt{2}\left(\sin^2\left(\frac{7\pi}{8}-2\alpha\right)-\sin^2\left(\frac{9\pi}{8}-2\alpha\right)\right)$ и найти его числовое значение при $\alpha=\pi/24$.

№86. $\dfrac{1}{tg^2\alpha}-\dfrac{2\cos 2\alpha}{1-\sin\left(2\alpha+\frac{\pi}{2}\right)}$

№87. $\dfrac{1-2\cos^2\alpha}{2tg\left(\alpha-\frac{\pi}{4}\right)\cdot\sin^2\left(\frac{\pi}{4}+\alpha\right)}$

III. Докажите тождества

№88. 1) $\cos\left(45^\circ + t\right) = \sin\left(45^\circ - t\right)$ 2) $\cos\left(45^\circ - t\right) = \sin\left(45^\circ + t\right)$

№89. 1) $ctg\left(45^\circ + t\right) = tg\left(45^\circ - t\right)$ 2) $ctg\left(45^\circ - t\right) = tg\left(45^\circ + t\right)$

№90. $\sin(t-\pi)\cdot tg(t+\pi) + \dfrac{1}{\cos(t-2\pi)} = \cos t$

№91. $\sin(2\pi + t)\cdot ctg(3\pi + t) - \cos(2\pi - t)\cdot tg(3\pi - t) = \sin t + \cos t$

№92. $\sin 395^\circ \cdot \sin 505^\circ + \cos 575^\circ \cdot \cos 865^\circ + tg 606^\circ \cdot tg 1104^\circ = 2$

№93. $\sin 405^\circ \cdot \cos 675^\circ + tg 562^\circ \cdot tg 788^\circ + \dfrac{1}{\cos 660^\circ}\cdot\dfrac{1}{\cos 1200^\circ} = -2{,}5$

№94. $\left(\sin\left(\dfrac{\pi}{2} - x\right) + \sin(\pi - x)\right)^2 + \left(\cos\left(\dfrac{3\pi}{2} - x\right) + \cos(2\pi - x)\right)^2 = 2$

№95. $\left(tg\dfrac{\pi}{4} + tg\left(\dfrac{\pi}{2} - x\right)\right)^2 + \left(ctg\dfrac{5\pi}{4} + ctg(\pi - x)\right)^2 = \dfrac{2}{\sin^2 x}$

№96. $\sin(2\pi - x)\cdot tg\left(\dfrac{3\pi}{2} - x\right) - \cos(x - \pi) - \sin(x - \pi) = \sin x$

№97. $\sin\left(\dfrac{\pi}{3} - t\right)\cdot tg\left(\dfrac{2\pi}{3} + t\right)\cdot\cos\left(\dfrac{5\pi}{3} + t\right) + tg(\pi + t)\cdot tg\left(\dfrac{3\pi}{2} - t\right) = \cos^2\left(\dfrac{\pi}{3} - t\right)$

№98. $\dfrac{\sin(x-\pi)\cdot\cos(x-2\pi)\cdot\sin(2\pi - x)}{\sin\left(\dfrac{\pi}{2} - x\right)\cdot ctg(\pi - x)\, ctg\left(\dfrac{3\pi}{2} + x\right)} = \sin^2 x$

№99. $\dfrac{\sin(\pi + x)\cdot\cos\left(\dfrac{3\pi}{2} - x\right)\cdot tg\left(x - \dfrac{\pi}{2}\right)}{\cos\left(\dfrac{\pi}{2} + x\right)\cdot tg(\pi + x)\cos\left(\dfrac{3\pi}{2} + x\right)} = ctg^2 x$

IV. Вычислите значение выражения при условии

№100. Вычислить $\sin\left(\frac{\pi}{2}+\alpha\right)$, если $\sin(\pi+\alpha)=0{,}8$ и $\alpha\in\left(-\frac{\pi}{2};0\right)$.

№101. Вычислить $\cos\left(\frac{3\pi}{2}+2\alpha\right)$, если $\sin\alpha-\cos\alpha=0{,}5$.

№102. Вычислить $\sin 2x$, если $\sin x+\sin(2{,}5\pi+x)=0{,}2$.

№103. Вычислить $\sin 2x$, если $\sin x+\sin(3{,}5\pi+x)=0{,}2$.

№104. Вычислить tg^2x+ctg^2x, если $tgx+tg\left(\frac{3\pi}{2}-x\right)=5$.

№105. Вычислить tg^2x+ctg^2x, если $tg(\pi-x)+ctgx=5$.

№106. Вычислить $\left(tg\left(\frac{5\pi}{4}+x\right)+tg\left(\frac{5\pi}{4}-x\right)\right)^{-1}$, если $tg\left(\frac{3\pi}{2}+x\right)=\frac{3}{4}$.

V. Разные задачи

№107. При каком значении *a* число $\frac{\pi}{4}$ является корнем уравнения $\sin^2 x+a\sin x\cos x-3\cos^2 x=0$

№108. При каком значении *a* число $\frac{3\pi}{4}$ является корнем уравнения $\sin^2 x+a\sin x\cos x+2\cos^2 x=0$

№109. При каком значении а выражение $\sin^2 x-\cos\left(\frac{\pi}{2}-x\right)\sin\left(\frac{a\pi}{4}-x\right)$ обращается в нуль при любом значении х.

№110. При каком значении а выражение $\cos^2 x-\sin\left(\frac{\pi}{2}-x\right)\sin\left(\frac{a\pi}{4}-x\right)$ обращается в нуль при любом значении х.

№111. Построить график функции $y=\sqrt{1+tg^2x}\cdot\dfrac{\cos^2(-x)\cdot\cos\left(\frac{\pi}{2}-x\right)}{tg\left(\frac{\pi}{2}-x\right)\cdot\sin^2(\pi+x)}$.

№112. Построить график функции $y=\sqrt{1+ctg^2x}\cdot\dfrac{ctg\left(\frac{\pi}{2}-x\right)\cdot\cos\left(\frac{3\pi}{2}+x\right)}{tg(\pi-x)}$.

▪ Решения

I. Вычислить:

		Ответы:
№1.	$\sin 540^\circ = \sin\left(360^\circ + 180^\circ\right) = \sin 180^\circ = 0$	0
№2.	$\sin 210^\circ = \sin(180^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$	$-\frac{1}{2}$
№3.	$\cos 330^\circ = \cos(360^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$
№4.	$\cos\left(-855^\circ\right) = \cos 855^\circ = \cos(720^\circ + 135^\circ) = \cos 135^\circ = \cos(180^\circ - 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
№5.	$tg 300^\circ = tg(360^\circ - 60^\circ) = tg(-60^\circ) = -tg 60^\circ = -\sqrt{3}$	$-\sqrt{3}$
№6.	$\cos 240^\circ = \cos(180^\circ + 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$	$-\frac{1}{2}$
№7.	$tg 3810^\circ = tg(360^\circ \cdot 10 + 210^\circ) = tg 210^\circ = tg(180^\circ + 30^\circ) = tg 30^\circ = \frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
№8.	$ctg 2,5\pi = ctg(2\pi + 0,5\pi) = ctg\frac{\pi}{2} = \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \frac{0}{1} = 0$	0
№9.	$\cos 2\frac{2}{3}\pi = \cos\left(3\pi - \frac{\pi}{3}\right) = \cos\left(2\pi + \pi - \frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$	$-\frac{1}{2}$
№10.	$\sin 7\frac{5}{6}\pi = \sin\left(8\pi - \frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$	$-\frac{1}{2}$
№11.	*Применим формулы косинус суммы и косинус разности* $\cos 105^\circ = \cos\left(45^\circ + 60^\circ\right) = \cos 45^\circ \cdot \cos 60^\circ - \sin 45^\circ \cdot \sin 60^\circ =$ $= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$	$\frac{\sqrt{2} - \sqrt{6}}{4}$
№12.	$\sin 285^\circ = \sin\left(270^\circ + 15^\circ\right) = -\cos 15^\circ = -\cos\left(45^\circ - 30^\circ\right) =$ $= -\left(\cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ\right) = -\left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right) = -\frac{\sqrt{6} + \sqrt{2}}{4}$	$-\frac{\sqrt{2} + \sqrt{6}}{4}$

№14.	$\left(\sin75^\circ+\sin100^\circ\right)\cdot\left(\sin260^\circ-\sin285^\circ\right)+\left(\sin165^\circ+\sin190^\circ\right)\cdot\left(\cos75^\circ-\cos100^\circ\right)=0$ 1) $\sin100^\circ=\sin\left(90^\circ+10^\circ\right)=\cos10^\circ$ 2) $\sin260^\circ=\sin\left(270^\circ-10^\circ\right)=-\cos10^\circ$ 3) $\sin285^\circ=\sin\left(270^\circ+15^\circ\right)=-\cos15^\circ$ 4) $\sin75^\circ=\cos15^\circ\rightarrow$ *Доп.уголы* $(75^\circ+15^\circ=90^\circ)$ 5) $\left(\cos15^\circ+\cos10^\circ\right)\left(-\cos10^\circ+\cos15^\circ\right)=\cos^2 15^\circ-\cos^2 10^\circ$ 6) $\sin165^\circ=\sin\left(180^\circ-15^\circ\right)=\sin15^\circ$ 7) $\sin190^\circ=\sin\left(180^\circ+10^\circ\right)=-\sin10^\circ$ 8) $\cos75^\circ=\sin15^\circ\rightarrow$ *Доп.уголы* $(75^\circ+15^\circ=90^\circ)$ 9) $\cos100^\circ=\cos\left(90^\circ+10^\circ\right)=-\sin10^\circ$ 10) $\left(\sin15^\circ-\sin10^\circ\right)\left(\sin15^\circ+\sin10^\circ\right)=\sin^2 15^\circ-\sin^2 10^\circ$ 11) $\cos^2 15^\circ-\cos^2 10^\circ+\sin^2 15^\circ-\sin^2 10^\circ=\left(\cos^2 15^\circ+\sin^2 15^\circ\right)-$ $-\left(\cos^2 10^\circ+\sin^2 10^\circ\right)=1-1=0$	0
№15.	$\left(\dfrac{tg^2 590^\circ}{\cos^2 320^\circ}+\dfrac{\sin111^\circ}{\cos159^\circ}\right)\cdot\left(\dfrac{\cos279^\circ}{\sin549^\circ}+\dfrac{ctg^2 950^\circ}{\sin^2 400^\circ}\right)=1$ 1) $tg^2 590^\circ=tg^2\left(360^\circ+180^\circ+50^\circ\right)=tg^2 50^\circ$ 2) $\cos^2 320^\circ=\cos^2\left(360^\circ-40^\circ\right)=\cos^2 40^\circ=\sin^2 50^\circ$ (*Доп.углы*) 3) $\sin111^\circ=\sin\left(90^\circ+21^\circ\right)=\cos21^\circ$ 4) $\cos159^\circ=\cos\left(180^\circ-21^\circ\right)=-\cos21^\circ$ 5) $\dfrac{tg^2 50^\circ}{\sin^2 50^\circ}+\dfrac{\cos21^\circ}{-\cos21^\circ}=\dfrac{\sin^2 50^\circ}{\cos^2 50^\circ\cdot\sin^2 50^\circ}-1=\dfrac{1}{\cos^2 50^\circ}-1=$ $=\dfrac{1-\cos^2 50^\circ}{\cos^2 50^\circ}=\dfrac{\sin^2 50^\circ}{\cos^2 50^\circ}=tg^2 50^\circ$ 6) $\cos279^\circ=\cos\left(270^\circ+9^\circ\right)=\sin9^\circ$ 7) $\sin549^\circ=\sin\left(360^\circ+180^\circ+9^\circ\right)=-\sin9^\circ$ 8) $ctg^2 950^\circ=ctg^2\left(360^\circ\cdot2+180^\circ+50^\circ\right)=ctg^2 50^\circ$ 9) $\sin^2 400^\circ=\sin^2\left(360^\circ+40^\circ\right)=\sin^2 40^\circ=\cos^2 50^\circ$ 10) $\dfrac{\sin9^\circ}{-\sin9^\circ}+\dfrac{ctg^2 50^\circ}{\cos^2 50^\circ}=-1+\dfrac{\cos^2 50^\circ}{\sin^2 50^\circ\cdot\cos^2 50^\circ}=\dfrac{1}{\sin^2 50^\circ}-1=$ $=\dfrac{1-\sin^2 50^\circ}{\sin^2 50^\circ}=\dfrac{\cos^2 50^\circ}{\sin^2 50^\circ}=ctg^2 50^\circ$ 11) $tg^2 50^\circ\cdot ctg^2 50^\circ=1$	1

№16.	$\sin 167^\circ \cdot \sin 107^\circ + \sin 257^\circ \cdot \sin 197^\circ = \frac{1}{2}$ 1) $\sin 167^\circ = \sin(180^\circ - 13^\circ) = \sin 13^\circ$ 2) $\sin 107^\circ = \sin(90^\circ + 17^\circ) = \cos 17^\circ$ 3) $\sin 257^\circ = \sin(270^\circ - 13^\circ) = -\cos 13^\circ$ 4) $\sin 197^\circ = \sin(180^\circ + 17^\circ) = -\sin 17^\circ$ 5) $\sin 13^\circ \cdot \cos 17^\circ + (-\cos 13^\circ)\cdot(-\sin 17^\circ) = \sin 13^\circ \cdot \cos 17^\circ + \cos 13^\circ \cdot \sin 17^\circ =$ $= \sin(13^\circ + 17^\circ) = \sin 30^\circ = \frac{1}{2}$	0,5
№17.	$\sin\frac{7\pi}{4} + \cos\frac{17\pi}{4} + tg\frac{19\pi}{4} + ctg\frac{7\pi}{4} = -2$ 1) $\sin\frac{7\pi}{4} = \sin\left(2\pi - \frac{\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$ 2) $\cos\frac{17\pi}{4} = \cos\left(4\pi + \frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ 3) $tg\frac{19\pi}{4} = tg\left(5\pi - \frac{\pi}{4}\right) = tg\left(-\frac{\pi}{4}\right) = -tg\frac{\pi}{4} = -1$ 4) $ctg\frac{7\pi}{4} = ctg\left(2\pi - \frac{\pi}{4}\right) = ctg\left(-\frac{\pi}{4}\right) = -ctg\frac{\pi}{4} = -1$ 5) $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 - 1 = -2$	-2
№18.	$\sin\left(-\frac{7\pi}{4}\right) + \cos\frac{7\pi}{4} + tg\frac{15\pi}{4} - ctg\left(-\frac{7\pi}{4}\right) = \sqrt{2} - 2$ 1) $\sin\left(-\frac{7\pi}{4}\right) = -\sin\frac{7\pi}{4} = -\sin\left(2\pi - \frac{\pi}{4}\right) = -\sin\left(-\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ 2) $\cos\frac{7\pi}{4} = \cos\left(2\pi - \frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ 3) $tg\frac{15\pi}{4} = tg\left(4\pi - \frac{\pi}{4}\right) = tg\left(-\frac{\pi}{4}\right) = -tg\frac{\pi}{4} = -1$ 4) $ctg\left(-\frac{7\pi}{4}\right) = -ctg\frac{7\pi}{4} = -ctg\left(2\pi - \frac{\pi}{4}\right) = -ctg\left(-\frac{\pi}{4}\right) = ctg\frac{\pi}{4} = 1$ 5) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 - 1 = \frac{2\sqrt{2}}{2} - 2 = \sqrt{2} - 2$	$\sqrt{2} - 2$
№19.	$\frac{1}{1 - 2\cos 30^\circ} + \frac{1}{1 + 2\sin 60^\circ} = \frac{1}{1 - 2\cdot\frac{\sqrt{3}}{2}} + \frac{1}{1 + 2\cdot\frac{\sqrt{3}}{2}} = \frac{1}{1 - \sqrt{3}} + \frac{1}{1 + \sqrt{3}} =$ $= \frac{1 + \sqrt{3} + 1 - \sqrt{3}}{1 - 3} = \frac{2}{-2} = -1$	-1

№20.

$$\frac{1}{tg60^\circ-1}-\frac{1}{ctg30^\circ+1}=\frac{1}{\sqrt{3}-1}-\frac{1}{\sqrt{3}+1}=\frac{\sqrt{3}+1-\sqrt{3}+1}{3-1}=1.$$

1

№21.

$$\frac{tg\frac{13\pi}{4}}{\cos\frac{7\pi}{4}+1}=\frac{tg\left(3\pi+\frac{\pi}{4}\right)}{\cos\left(2\pi-\frac{\pi}{4}\right)+1}=\frac{tg\frac{\pi}{4}}{\cos\left(-\frac{\pi}{4}\right)+1}=\frac{1}{\cos\frac{\pi}{4}+1}=\frac{1}{\frac{\sqrt{2}}{2}+1}=\frac{2}{\sqrt{2}+2}=$$

$$=\frac{2\left(\sqrt{2}-2\right)}{\left(\sqrt{2}+2\right)\left(\sqrt{2}-2\right)}=\frac{2\left(\sqrt{2}-2\right)}{2-4}=\frac{2\left(\sqrt{2}-2\right)}{-2}=-\left(\sqrt{2}-2\right)=2-\sqrt{2}$$

$2-\sqrt{2}$

№22.

$$\frac{ctg\left(-\frac{7\pi}{4}\right)}{\sin\frac{13\pi}{4}+1}=\frac{-ctg\left(2\pi-\frac{\pi}{4}\right)}{\sin\left(3\pi+\frac{\pi}{4}\right)+1}=\frac{-ctg\left(-\frac{\pi}{4}\right)}{\sin\left(\pi+\frac{\pi}{4}\right)+1}=\frac{ctg\frac{\pi}{4}}{-\sin\frac{\pi}{4}+1}=\frac{1}{1-\frac{\sqrt{2}}{2}}=$$

$$=\frac{2}{2-\sqrt{2}}=\frac{2\cdot\left(2+\sqrt{2}\right)}{\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)}=\frac{2\left(2+\sqrt{2}\right)}{4-2}=2+\sqrt{2}$$

$2+\sqrt{2}$

№23.

$$\frac{1}{\cos1110^\circ+\cos2220^\circ+\cos3330^\circ}$$

1) $\cos1110^\circ=\cos\left(360^\circ\cdot3+30^\circ\right)=\cos30^\circ=\frac{\sqrt{3}}{2}$

2) $\cos2220^\circ=\cos\left(2\cdot1110^\circ\right)=\cos\left(2\cdot\left(360^\circ\cdot3+30^\circ\right)\right)=\cos\left(360^\circ\cdot6+60^\circ\right)=\cos60^\circ=\frac{1}{2}$

3) $\cos3330^\circ=\cos\left(3\cdot1110^\circ\right)=\cos\left(3\cdot\left(360^\circ\cdot3+30^\circ\right)\right)=\cos\left(360^\circ\cdot9+90^\circ\right)=\cos90^\circ=0$

4) $\frac{1}{\frac{\sqrt{3}}{2}+\frac{1}{2}+0}=\frac{2}{1+\sqrt{3}}=\frac{2\cdot\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}=\frac{2\left(\sqrt{3}-1\right)}{3-1}=\sqrt{3}-1$

$\sqrt{3}-1$

№24.

$$\left(\cos1140^\circ+\sin2280^\circ+\sin3420^\circ\right)^{-1}$$

1) $\cos1140^\circ=\cos\left(360^\circ\cdot3+60^\circ\right)=\cos60^\circ=\frac{1}{2}$

2) $\sin2280^\circ=\sin\left(2\cdot1140^\circ\right)=\sin\left(2\cdot\left(360^\circ\cdot3+60^\circ\right)\right)=$

$=\sin\left(360^\circ\cdot6+2\cdot60^\circ\right)=\sin120^\circ=\sin\left(180^\circ-60^\circ\right)=\sin60^\circ=\frac{\sqrt{3}}{2}$

3) $\sin3420^\circ=\sin\left(3\cdot1140^\circ\right)=\sin\left(3\cdot\left(360^\circ\cdot3+60^\circ\right)\right)=\sin\left(360^\circ\cdot9+180^\circ\right)=\sin180^\circ=0$

4) $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}+0\right)^{-1}=\left(\frac{1+\sqrt{3}}{2}\right)^{-1}=\frac{2}{1+\sqrt{3}}=\frac{2\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}=\frac{2\left(\sqrt{3}-1\right)}{2}=\sqrt{3}-1$

$\sqrt{3}-1$

№25.	$tg615^\circ + tg375^\circ = 4$ 1) $tg615^\circ = tg\left(360^\circ \cdot 2 - 105^\circ\right) = tg\left(-105^\circ\right) = -tg105^\circ = -tg\left(90^\circ + 15^\circ\right) =$ $= ctg15^\circ$ 2) $tg375^\circ = tg\left(360^\circ + 15^\circ\right) = tg15^\circ$ 3) $ctg15^\circ + tg15^\circ = \frac{\cos 15^\circ}{\sin 15^\circ} + \frac{\sin 15^\circ}{\cos 15^\circ} = \frac{\cos^2 15^\circ + \sin^2 15^\circ}{\sin 15^\circ \cdot \cos 15^\circ} =$ $= \frac{1}{\sin 15^\circ \cdot \cos 15^\circ} = \frac{2}{2 \cdot \sin 15^\circ \cdot \cos 15^\circ} = \frac{2}{\sin 30^2} = \frac{2}{\frac{1}{2}} = 4$	4
№26.	$ctg\frac{13\pi}{12} - ctg\frac{5\pi}{12} = 2\sqrt{3}$ 1) $ctg\frac{13\pi}{12} = ctg\left(\pi + \frac{\pi}{12}\right) = ctg\frac{\pi}{12}$ *Заметим, что* $\frac{5\pi}{12} + \frac{\pi}{12} = \frac{6\pi}{12} = \frac{\pi}{2}$, *т.к. углы дополняют* *друг друга до* $\frac{\pi}{2} \Rightarrow ctg\frac{5\pi}{12} = tg\frac{\pi}{12}$ 2) $ctg\frac{\pi}{12} - tg\frac{\pi}{12} = \frac{\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}} - \frac{\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}} = \frac{\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}}{\sin\frac{\pi}{12} \cdot \cos\frac{\pi}{12}} =$ $= \frac{\cos 2 \cdot \frac{\pi}{12}}{\sin\frac{\pi}{12} \cdot \cos\frac{\pi}{12}} = \frac{2 \cdot \cos\frac{\pi}{6}}{2 \cdot \sin\frac{\pi}{12} \cdot \cos\frac{\pi}{12}} = \frac{2 \cdot \frac{\sqrt{3}}{2}}{\sin\frac{\pi}{6}} = \frac{\sqrt{3}}{\frac{1}{2}} = 2\sqrt{3}$	$2\sqrt{3}$
№27.	$\left(\sin\frac{7\pi}{18} - \sin\frac{\pi}{9}\right) : \left(\cos\frac{7\pi}{18} - \cos\frac{\pi}{9}\right) = -1$ *Заметим, что* $\frac{7\pi}{18} + \frac{\pi}{9} = \frac{7\pi + 2\pi}{18} = \frac{9\pi}{18} = \frac{\pi}{2}$, *т.к. углы* $\frac{7\pi}{18}$ *и* $\frac{\pi}{9}$ – *дополнительные* $\Rightarrow \sin\frac{7\pi}{18} = \cos\frac{\pi}{9}$ $\left(\cos\frac{\pi}{9} - \sin\frac{\pi}{9}\right) : \left(\sin\frac{\pi}{9} - \cos\frac{\pi}{9}\right) = -1$	-1
№28.	$\frac{6\sin 35^\circ \cdot \sin 55^\circ}{\cos 20^\circ} = 3$ 1) $\sin 55^\circ = \cos 35^\circ$, *т.к.* $35^\circ + 55^\circ = 90^\circ$ 2) $6 \cdot \sin 35^\circ \cdot \cos 35^\circ = 3 \cdot 2 \cdot \sin 35^\circ \cdot \cos 35^\circ = 3 \cdot \sin 70^\circ =$ $= 3 \cdot \cos 20^\circ$ (*т.к.* $70^\circ + 20^\circ = 90^\circ$) 3) $\frac{3 \cdot \cos 20^\circ}{\cos 20^\circ} = 3$	3

№29.	$\dfrac{\sin 54^\circ}{\cos 63^\circ \cdot \sin 117^\circ} = 1$ 1) $\sin 117^\circ = \sin\left(90^\circ + 27^\circ\right) = \cos 27^\circ$ 2) $\cos 63^\circ = \sin 27^\circ, \textit{т.к.}\ 27^\circ + 63^\circ = 90^\circ$ 3) $\sin 54^\circ = \sin 2 \cdot 27^\circ = 2\sin 27^\circ \cdot \cos 27^\circ$ 4) $\dfrac{2\sin 27^\circ \cdot \cos 27^\circ}{\sin 27^\circ \cdot \cos 27^\circ} = 2$	2
№30.	$2\sin^2 \dfrac{\pi}{12} - 1 = -\left(1 - 2\sin^2 \dfrac{\pi}{12}\right) = -\cos 2 \cdot \dfrac{\pi}{12} = -\cos \dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{2}$	$-\dfrac{\sqrt{3}}{2}$
№31.	$\cos 195^\circ \cdot \cos 105^\circ + \sin 105^\circ \cdot \cos 75^\circ$ 1) $\cos 195^\circ = \cos\left(180^\circ + 15^\circ\right) = -\cos 15^\circ$ 2) $\cos 105^\circ = \cos\left(90^\circ + 15^\circ\right) = -\sin 15^\circ$ 3) $\sin 105^\circ = \sin\left(90^\circ + 15^\circ\right) = \cos 15^\circ$ 4) $\cos 75^\circ = \sin 15^\circ$ 5) $-\cos 15^\circ \cdot \left(-\sin 15^\circ\right) + \cos 15^\circ \cdot \sin 15^\circ = 2\sin 15^\circ \cos 15^\circ = \sin 30^\circ = \dfrac{1}{2}$	0,5
№32.	$\sin 23^\circ \cdot \sin 53^\circ + \sin 67^\circ \cdot \sin 37^\circ$ 1) $\sin 67^\circ = \cos 23^\circ, \quad \sin 37^\circ = \cos 53^\circ$ 2) $\sin 23^\circ \cdot \sin 53^\circ + \cos 23^\circ \cdot \cos 53^\circ = \cos\left(53^\circ - 23^\circ\right) = \cos 30^\circ = \dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{3}}{2}$
№33.	$ctg5^\circ \cdot ctg10^\circ \cdot ctg15^\circ \cdot \ldots \cdot ctg85^\circ = 1$ 1) $ctg85^\circ = tg5^\circ, \textit{т.к.}\ 5^\circ + 85^\circ = 90^\circ$ $ctg80^\circ = tg10^\circ$ *и т.д.* . 2) $ctg5^\circ \cdot tg5^\circ = 1$ $ctg10^\circ \cdot tg10^\circ = 1$ *и т.д.*	1
№34.	$tg3^\circ \cdot tg6^\circ \cdot tg9^\circ \cdot \ldots \cdot tg87^\circ = 1$ 1) $tg87^\circ = ctg3^\circ, \textit{т.к.}\ 3^\circ + 87^\circ = 90^\circ$ $tg84^\circ = ctg6^\circ$ *и т.д.* 2) $tg3^\circ \cdot ctg3^\circ = 1$ $tg6^\circ \cdot ctg6^\circ = 1$ *и т.д.*	1
№35.	$\cos 20^\circ + \cos 40^\circ + \cos 60^\circ + \ldots + \cos 160^\circ + \cos 180^\circ = -1$ 1) $\cos 160^\circ = \cos\left(180^\circ - 20^\circ\right) = -\cos 20^\circ$ 2) $\cos 140^\circ = \cos\left(180^\circ - 40^\circ\right) = -\cos 40^\circ$ 3) $\cos 20^\circ + \cos 40^\circ + \cos 60^\circ + \ldots + (-\cos 60^\circ) + (-\cos 40^\circ) + (-\cos 20^\circ) + (-1) = -1$	-1

№36.	$tg20^\circ + tg40^\circ + tg60^\circ + \ldots + tg160^\circ + tg180^\circ = 0$ 1) $tg160^\circ = tg\left(180^\circ - 20^\circ\right) = -tg20^\circ$ $tg140^\circ = tg\left(180^\circ - 40^\circ\right) = -tg40^\circ$ 2) $tg20^\circ + tg40^\circ + tg60^\circ + \ldots + (-tg60^\circ) + (-tg40^\circ) + (-tg20^\circ) + 0 = 0$	0
№37.	$\sin 0^\circ + \sin 1^\circ + \sin 2^\circ + \ldots + \sin 359^\circ + \sin 360^\circ = 0$ 1) $\sin 359^\circ = \sin(360^\circ - 1^\circ) = \sin(-1^\circ) = -\sin 1^\circ$ 2) $0 + \sin 1^\circ + \sin 2^\circ + \ldots + (-\sin 2^\circ) + (-\sin 1^\circ) + 0 = 0$	0
№38.	$ctg15^\circ + ctg30^\circ + ctg45^\circ + \ldots + ctg165^\circ$ 1) $ctg165^\circ = ctg\left(180^\circ - 15^\circ\right) = ctg\left(-15^\circ\right) = -ctg15^\circ$ 2) $ctg15^\circ + ctg30^\circ + ctg45^\circ + \ldots + \left(-ctg45^\circ\right) + \left(-ctg30^\circ\right) + \left(-ctg15^\circ\right) = 0$	0
№39.	$\sin^4 \frac{7\pi}{8} + \cos^4 \frac{\pi}{8}$ $\boxed{\begin{array}{l} \sin^4 t + \cos^4 t = 1 - 2\sin^2 t \cdot \cos^2 t \\ \sin^2 2t = \left(2\sin t \cdot \cos t\right)^2 = 4\sin^2 t \cdot \cos^2 t \end{array}}$ 1) $\sin \frac{7\pi}{8} = \sin\left(\pi - \frac{\pi}{8}\right) = \sin \frac{\pi}{8}$ 2) $\sin^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} = 1 - 2\sin^2 \frac{\pi}{8} \cdot \cos^2 \frac{\pi}{8} = 1 - \frac{4\sin^2 \frac{\pi}{8} \cdot \cos^2 \frac{\pi}{8}}{2} =$ $= 1 - \frac{\sin^2\left(2 \cdot \frac{\pi}{8}\right)}{2} = 1 - \frac{\sin^2 \frac{\pi}{4}}{2} = 1 - \frac{\left(\frac{\sqrt{2}}{2}\right)^2}{2} = 1 - \frac{1}{4} = \frac{3}{4} = 0,75$	0,75
№40.	$\left(\sin \frac{5\pi}{8} + \cos \frac{3\pi}{8}\right)^2 = 1 + \frac{\sqrt{2}}{2}$ 1) $\sin \frac{5\pi}{8} = \sin\left(\pi - \frac{3\pi}{8}\right) = \sin \frac{3\pi}{8}$ 2) $\left(\sin \frac{3\pi}{8} + \cos \frac{3\pi}{8}\right)^2 = \sin^2 \frac{3\pi}{8} + 2 \cdot \sin \frac{3\pi}{8} \cdot \cos \frac{3\pi}{8} + \cos^2 \frac{3\pi}{8} = 1 + \sin 2 \cdot \frac{3\pi}{8} =$ $= 1 + \sin \frac{3\pi}{4} = 1 + \sin\left(\pi - \frac{\pi}{4}\right) = 1 + \sin \frac{\pi}{4} = 1 + \frac{\sqrt{2}}{2}$	$1+\frac{\sqrt{2}}{2}$

№41.	$\sin^6\frac{\pi}{8}+\cos^6\frac{7\pi}{8}$ $\boxed{\sin^6 t+\cos^6 t=1-3\sin^2 t\cdot\cos^2 t \quad \sin^2 2t=4\sin^2 t\cdot\cos^2 t}$ 1) $\cos^6\frac{7\pi}{8}=\cos^6\left(\pi-\frac{\pi}{8}\right)=\cos^6\frac{\pi}{8}$ 2) $\sin^6\frac{\pi}{8}+\cos^6\frac{\pi}{8}=1-3\cdot\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}=1-\frac{3\cdot\left(4\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}\right)}{4}=$ $=1-\frac{3\cdot\sin^2\left(2\cdot\frac{\pi}{8}\right)}{4}=1-\frac{3\cdot\sin^2\frac{\pi}{4}}{4}=1-\frac{3\cdot\left(\frac{\sqrt{2}}{2}\right)^2}{4}=1-\frac{3\cdot\frac{1}{2}}{4}=1-\frac{3}{8}=\frac{5}{8}$	5/8
№42.	$\sin^6\frac{\pi}{8}-\cos^6\frac{7\pi}{8}$ $\boxed{a^3-b^3=(a-b)(a^2+ab+b^2) \quad \sin^4 t+\cos^4 t=1-2\sin^2 t\cdot\cos^2 t}$ 1) $\cos^6\frac{7\pi}{8}=\cos^6\frac{\pi}{8}$ 2) $\left(\sin^2\frac{\pi}{8}\right)^3-\left(\cos^2\frac{\pi}{8}\right)^3=\left(\sin^2\frac{\pi}{8}-\cos^2\frac{\pi}{8}\right)\left(\sin^4\frac{\pi}{8}+\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}+\cos^4\frac{\pi}{8}\right)=$ $=-\cos 2\cdot\frac{\pi}{8}\left(1-2\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}\right)=-\cos\frac{\pi}{4}\cdot\left(1-\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}\right)=$ $=-\frac{\sqrt{2}}{2}\left(1-\frac{4\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}}{4}\right)=-\frac{\sqrt{2}}{2}\cdot\left(1-\frac{\sin^2\frac{\pi}{4}}{4}\right)=-\frac{\sqrt{2}}{2}\left(1-\frac{1}{8}\right)=-\frac{\sqrt{2}}{2}\cdot\frac{7}{8}=-\frac{7\sqrt{2}}{16}$	$-\frac{7\sqrt{2}}{16}$
№43.	$\frac{\sin 70^\circ+\cos 40^\circ}{\sin 280^\circ}=-\sqrt{3}$ $\boxed{\cos x+\cos y=2\cdot\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}}$ 1) $\sin 280^\circ=\sin(270^\circ+10^\circ)=-\cos 10^\circ$ 2) $\sin 70^\circ=\cos 20^\circ$ 3) $\cos 20^\circ+\cos 40^\circ=2\cdot\cos\frac{20^\circ+40^\circ}{2}\cdot\cos\frac{20^\circ-40^\circ}{2}=2\cdot\cos 30^\circ\cdot\cos 10^\circ=$ $=2\cdot\frac{\sqrt{3}}{2}\cdot\cos 10^\circ=\sqrt{3}\cdot\cos 10^\circ$ 4) $\frac{\sqrt{3}\cdot\cos 10^\circ}{-\cos 10^\circ}=-\sqrt{3}$ 5) $tg300^\circ=(tg360^\circ-60^\circ)=tg(-60^\circ)=-\sqrt{3}$	$tg300^\circ$

№44.	$\dfrac{\cos 20^\circ - \sin 50^\circ}{\cos 280^\circ} = 1$ $\boxed{\sin x - \sin y = 2\cdot\sin\dfrac{x-y}{2}\cdot\cos\dfrac{x-y}{2}}$ 1) $\cos 20^\circ = \sin 70^\circ$ 2) $\sin 70^\circ - \sin 50^\circ = 2\cdot\sin\dfrac{70^\circ - 50^\circ}{2}\cdot\cos\dfrac{70^\circ + 50^\circ}{2} = 2\cdot\sin 10^\circ\cdot\cos 60^\circ =$ $= 2\cdot\dfrac{1}{2}\cdot\sin 10^\circ = \sin 10^\circ$ 3) $\cos 280^\circ = \cos\left(270^\circ + 10^\circ\right) = \sin 10^\circ$ 4) $\dfrac{\sin 10^\circ}{\sin 10^\circ} = 1$	1
№45.	$\dfrac{\sin 50^\circ + 2\sin 10^\circ}{\cos 50^\circ} = \dfrac{\sin 50^\circ + \sin 10^\circ + \sin 10^\circ}{\cos 50^\circ} = \sqrt{3}$ $\boxed{\sin x + \sin y = 2\cdot\sin\dfrac{x+y}{2}\cdot\cos\dfrac{x+y}{2}}$ 1) $\sin 50^\circ + \sin 10^\circ = 2\cdot\sin\dfrac{50^\circ + 10^\circ}{2}\cdot\cos\dfrac{50^\circ - 10^\circ}{2} = 2\cdot\sin 30^\circ\cdot\cos 20^\circ =$ $= 2\cdot\dfrac{1}{2}\cdot\cos 20^\circ = \cos 20^\circ$ 2) $\cos 20^\circ + \sin 10^\circ = \sin 70^\circ + \sin 10^\circ = 2\cdot\sin\dfrac{70^\circ + 10^\circ}{2}\cdot\cos\dfrac{70^\circ - 10^\circ}{2} =$ $= 2\cdot\sin 40^\circ\cdot\cos 30^\circ = 2\cdot\cos 50^\circ\cdot\dfrac{\sqrt{3}}{2} = \sqrt{3}\cdot\cos 50^\circ$ 3) $\dfrac{\sqrt{3}\cdot\cos 50^\circ}{\cos 50^\circ} = \sqrt{3}$	$\sqrt{3}$
№46.	$\dfrac{\cos 35^\circ + 2\cos 85^\circ}{\sqrt{3}\cos 55^\circ} = \dfrac{\cos 35^\circ + \cos 85^\circ + \cos 85^\circ}{\sqrt{3}\cdot\cos 55^\circ} = 1$ $\boxed{\cos x + \cos y = 2\cdot\cos\dfrac{x+y}{2}\cdot\cos\dfrac{x-y}{2}}$ 1) $\cos 35^\circ + \cos 85^\circ = 2\cdot\cos\dfrac{35^\circ + 85^\circ}{2}\cdot\cos\dfrac{35^\circ - 85^\circ}{2} = 2\cdot\cos 60^\circ\cdot\cos 25^\circ =$ $= 2\cdot\dfrac{1}{2}\cdot\cos 25^\circ = \cos 25^\circ$ 2) $\cos 25^\circ + \cos 85^\circ = 2\cdot\cos\dfrac{25^\circ + 85^\circ}{2}\cdot\cos\dfrac{25^\circ - 85^\circ}{2} = 2\cdot\cos 55^\circ\cdot\cos 30^\circ =$ $= 2\cdot\dfrac{\sqrt{3}}{2}\cdot\cos 55^\circ = \sqrt{3}\cdot\cos 55^\circ$ 3) $\dfrac{\sqrt{3}\cdot\cos 55^\circ}{\sqrt{3}\cdot\cos 55^\circ} = 1$	1

№47.	$\dfrac{\sin 50^\circ \cdot \cos 12^\circ - \sin 40^\circ \cdot \cos 78^\circ}{\cos 68^\circ - \sqrt{3}\sin 68^\circ} = -0,5$ 1) $\sin 50^\circ = \cos 40^\circ$ 2) $\cos 78^\circ = \sin 12^\circ$ 3) $\cos 40^\circ \cdot \cos 12^\circ - \sin 40^\circ \cdot \sin 12^\circ = \cos\left(40^\circ + 12^\circ\right) = \cos 52^\circ$ 4) $\cos 68^\circ - \sqrt{3}\sin 68^\circ = 2\cdot\left(\dfrac{1}{2}\cdot\cos 68^\circ - \dfrac{\sqrt{3}}{2}\sin 68^\circ\right) =$ $= 2\cdot\left(\cos 60^\circ \cdot \cos 68^\circ - \sin 60^\circ \cdot \sin 68^\circ\right) = 2\cdot\cos\left(60^\circ + 68^\circ\right) =$ $= 2\cdot\cos 128^\circ = 2\cdot\cos\left(180^\circ - 52^\circ\right) = -2\cdot\cos 52^\circ$ 5) $\dfrac{\cos 52^\circ}{-2\cos 52^\circ} = -\dfrac{1}{2} = -0,5$	-0,5
№48.	$\dfrac{2\sin^2 70^\circ - 1}{2ctg115^\circ \cdot \cos^2 155^\circ}$ 1) $2\sin^2 70^\circ - 1 = -\left(1 - 2\sin^2 70^\circ\right) = -\cos 2\cdot 70^\circ = -\cos 140^\circ = -\cos\left(180^\circ - 40^\circ\right) = \cos 40^\circ$ 2) $ctg115^\circ = ctg\left(90^\circ + 25^\circ\right) = -tg25^\circ$ 3) $\cos^2 155^\circ = \cos^2\left(180^\circ - 25^\circ\right) = \cos^2 25^\circ$ 4) $\dfrac{\cos 40^\circ}{-2\cdot tg25^\circ \cdot \cos^2 25^\circ} = -\dfrac{\sin 50^\circ}{2\cdot\dfrac{\sin 25^\circ}{\cos 25^\circ}\cdot\cos^2 25^\circ} = -\dfrac{\sin 50^\circ}{2\sin 25^\circ \cdot \cos 25^\circ} = -\dfrac{\sin 50^\circ}{\sin 50^\circ} = -1$	-1
№49.	$8\left(\sqrt{3} - \sqrt{2}\right)\sin 52^\circ 30' \cdot \cos 7^\circ 30'$ $\boxed{\sin x \cdot \cos y = \dfrac{1}{2}\left(\sin\left(x + y\right) + \sin\left(x - y\right)\right)}$ $8\left(\sqrt{3} - \sqrt{2}\right)\sin 52^\circ 30' \cdot \cos 7^\circ 30' = 8\left(\sqrt{3} - \sqrt{2}\right)\cdot\dfrac{1}{2}\left(\sin\left(52^\circ 30' + 7^\circ 30'\right) + \sin\left(52^\circ 30' - 7^\circ 30'\right)\right) =$ $= 4\left(\sqrt{3} - \sqrt{2}\right)\cdot\left(\sin 60^\circ + \sin 45^\circ\right) = 4\left(\sqrt{3} - \sqrt{2}\right)\cdot\left(\dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{2}}{2}\right) =$ $= \dfrac{4}{2}\cdot\left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{3} + \sqrt{2}\right) = 2$	2
№50.	$\sin^2\dfrac{\pi}{13} + \sin^2\dfrac{11\pi}{26}$ $т.к.\ \dfrac{\pi}{13} + \dfrac{11\pi}{26} = \dfrac{2\pi + 11\pi}{26} = \dfrac{13\pi}{26} = \dfrac{\pi}{2},\ то\ \sin\dfrac{11\pi}{26} = \cos\dfrac{\pi}{13}$ $\sin^2\dfrac{\pi}{13} + \cos^2\dfrac{\pi}{13} = 1$	1
№51.	$\dfrac{\sqrt{2\left(1 - \sin 82^\circ\right)}}{\sin 4^\circ} = \dfrac{\sqrt{2\left(1 - \cos 8^\circ\right)}}{\sin 4^\circ} = \dfrac{\sqrt{2\cdot 2\cdot\sin^2 4^\circ}}{\sin 4^\circ} = \dfrac{2\cdot\sin 4^\circ}{\sin 4^\circ} = 2$ $\boxed{1 - \cos 2x = 2\sin^2 x}$	32

№52.	$\dfrac{\sqrt{3}+tg\dfrac{11\pi}{12}}{1+\sqrt{3}tg\dfrac{\pi}{12}}=1$ $\boxed{tg(x-y)=\dfrac{tgx-tgy}{1+tgx\cdot tgy}}$ 1) $tg\dfrac{11\pi}{12}=tg\left(\pi-\dfrac{\pi}{12}\right)=-tg\dfrac{\pi}{12}$. 2) $\sqrt{3}=tg\dfrac{\pi}{3}$ 3) $\dfrac{tg\dfrac{\pi}{3}-tg\dfrac{\pi}{12}}{1+tg\dfrac{\pi}{3}\cdot tg\dfrac{\pi}{12}}=tg\left(\dfrac{\pi}{3}-\dfrac{\pi}{12}\right)=tg\dfrac{4\pi-\pi}{12}=tg\dfrac{3\pi}{12}=tg\dfrac{\pi}{4}=1$	1
№53.	$\dfrac{\dfrac{1}{\sqrt{3}}+tg\dfrac{13\pi}{12}}{\sqrt{3}-tg\dfrac{\pi}{12}}=\dfrac{1}{\sqrt{3}}$ $\boxed{tg(x+y)=\dfrac{tgx+tgy}{1-tgx\cdot tgy}}$ 1) $tg\dfrac{13\pi}{12}=tg\left(\pi+\dfrac{\pi}{12}\right)=tg\dfrac{\pi}{12}$ 2) $\dfrac{1}{\sqrt{3}}=tg\dfrac{\pi}{6};\ \sqrt{3}=\dfrac{1}{tg\dfrac{\pi}{6}}$ 3) $\dfrac{tg\dfrac{\pi}{6}+tg\dfrac{\pi}{12}}{\dfrac{1}{tg\dfrac{\pi}{6}}-tg\dfrac{\pi}{12}}=\dfrac{\left(tg\dfrac{\pi}{6}+tg\dfrac{\pi}{12}\right)\cdot tg\dfrac{\pi}{6}}{1-tg\dfrac{\pi}{6}\cdot tg\dfrac{\pi}{12}}=tg\left(\dfrac{\pi}{6}+\dfrac{\pi}{12}\right)\cdot tg\dfrac{\pi}{6}=tg\dfrac{\pi}{4}\cdot tg\dfrac{\pi}{6}=1\cdot\dfrac{1}{\sqrt{3}}=\dfrac{1}{\sqrt{3}}$.	$\dfrac{1}{\sqrt{3}}$
№54.	$\left(\left(tg^2\dfrac{7\pi}{24}-tg^2\dfrac{\pi}{24}\right):\left(1-tg^2\dfrac{7\pi}{24}\cdot tg^2\dfrac{\pi}{24}\right)\right)^2=\left(\dfrac{\left(tg\dfrac{7\pi}{24}-tg\dfrac{\pi}{24}\right)\left(tg\dfrac{7\pi}{24}+tg\dfrac{\pi}{24}\right)}{\left(1+tg\dfrac{7\pi}{24}\cdot tg\dfrac{\pi}{24}\right)\left(1-tg\dfrac{7\pi}{24}\cdot tg\dfrac{\pi}{24}\right)}\right)^2=$ $=\left(tg\left(\dfrac{7\pi}{24}-\dfrac{\pi}{24}\right)\cdot tg\left(\dfrac{7\pi}{24}+\dfrac{\pi}{24}\right)\right)^2=\left(tg\dfrac{\pi}{4}\cdot tg\dfrac{\pi}{3}\right)^2=\left(1\cdot\sqrt{3}\right)^2=3$	3

№55.	$\sqrt{3}\left(tg\frac{51\pi}{36}-tg\frac{13\pi}{12}\right)=6$ 1) $tg\frac{51\pi}{36}=tg\left(\pi+\frac{15\pi}{36}\right)=tg\frac{15\pi}{36}$ 2) $tg\frac{13\pi}{12}=tg\left(\pi+\frac{\pi}{12}\right)=tg\frac{\pi}{12}$ 3) $т.к.\ \frac{15\pi}{36}+\frac{\pi}{12}=\frac{15\pi+3\pi}{36}=\frac{18\pi}{36}=\frac{\pi}{2}, то\ tg\frac{15\pi}{36}=ctg\frac{\pi}{12}$ 4) $ctg\frac{\pi}{12}-tg\frac{\pi}{12}=\frac{\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}}-\frac{\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}}=\frac{\cos^2\frac{\pi}{12}-\sin^2\frac{\pi}{12}}{\sin\frac{\pi}{12}\cdot\cos\frac{\pi}{12}}=$ $=\frac{2\cdot\cos 2\cdot\frac{\pi}{12}}{2\cdot\sin\frac{\pi}{12}\cdot\cos\frac{\pi}{12}}=\frac{2\cdot\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}=2\cdot ctg\frac{\pi}{6}=2\sqrt{3}$ 5) $\sqrt{3}\cdot 2\cdot\sqrt{3}=6$	6
№56.	$\frac{\cos 2,9\pi\cdot tg2,4\pi\cdot tg1,1\pi}{\cos 0,9\pi}=1$ 1) $\cos 2,9\pi=\cos(2\pi+0,9\pi)=\cos 0,9\pi$ 2) $tg2,4\pi=tg(2\pi+0,4\pi)=tg0,4\pi=tg(0,5\pi-0,1\pi)=ctg0,1\pi$ 3) $tg1,1\pi=tg(\pi+0,1\pi)=tg0,1\pi$ 4) $\frac{\cos 0,9\pi\cdot ctg0,1\pi\cdot tg0,1\pi}{\cos 0,9\pi}=1$	1
№57.	$\frac{\cos 0,9\pi\cdot ctg0,4\pi\cdot ctg2,1\pi}{\cos 2,1\pi}=-1$ 1) $\cos 0,9\pi=\cos(\pi-0,1\pi)=-\cos 0,1\pi$ 2) $ctg0,4\pi=ctg(0,5\pi-0,1\pi)=tg0,1\pi$ 3) $ctg2,1\pi=ctg(2\pi+0,1\pi)=ctg0,1\pi$ 4) $\cos 2,1\pi=\cos(2\pi+0,1\pi)=\cos 0,1\pi$ 5) $\frac{-\cos 0,1\pi\cdot tg0,1\pi\cdot ctg0,1\pi}{\cos 0,1\pi}=-1$	-1

II. Упростить выражения:

№58.	$\sin\left(\alpha-\frac{\pi}{2}\right)\cdot\cos\left(\alpha-\frac{3\pi}{2}\right)\cdot\cos(\alpha-\pi)=-\sin\left(\frac{\pi}{2}-\alpha\right)\cdot\cos\left(\frac{3\pi}{2}-\alpha\right)\cdot\cos(\pi-\alpha)=$ $=-\cos\alpha\cdot(-\sin\alpha)\cdot(-\cos\alpha)=-\cos^2\alpha\cdot\sin\alpha$	$-\sin\alpha\cdot\cos^2\alpha$
№59.	$\cos\left(\alpha-\frac{\pi}{2}\right)\cdot tg\left(\alpha-\frac{3\pi}{2}\right)\cdot tg(8\pi-\alpha)=\cos\left(\frac{\pi}{2}-\alpha\right)\cdot\left(-tg\left(\frac{3\pi}{2}-\alpha\right)\right)\cdot tg(-\alpha)=$ $=-\sin\alpha\cdot ctg\alpha\cdot(-tg\alpha)=\sin\alpha\cdot ctg\alpha\cdot tg\alpha=\sin\alpha$	$\sin\alpha$
№60.	$\sin\left(\alpha-\frac{\pi}{2}\right)\cdot\cos\left(\alpha-\frac{3\pi}{2}\right)\cdot\cos(\alpha-\pi)=\frac{\sin\left(\frac{3\pi}{2}+\alpha\right)\cdot tg\left(\alpha-\frac{\pi}{2}\right)}{-\cos\alpha}=$ $=\frac{-\cos\alpha\cdot\left(-tg\left(\frac{\pi}{2}-\alpha\right)\right)}{-\cos\alpha}=-ctg\alpha$	$-ctg\alpha$
№61.	$\frac{\cos\frac{5\pi-2\alpha}{2}\cdot ctg\frac{2\alpha-3\pi}{2}}{\sin(\alpha-3\pi)}=\frac{\cos\left(\frac{5\pi}{2}-\alpha\right)\cdot ctg\left(\alpha-\frac{3\pi}{2}\right)}{-\sin(2\pi+\pi-\alpha)}=$ $=\frac{\cos\left(2\pi+\frac{\pi}{2}-\alpha\right)\cdot\left(-ctg\left(\frac{3\pi}{2}-\alpha\right)\right)}{-\sin(\pi-\alpha)}=\frac{-\cos\left(\frac{\pi}{2}-\alpha\right)\cdot tg\alpha}{-\sin\alpha}=$ $=\frac{\sin\alpha\cdot tg\alpha}{\sin\alpha}=tg\alpha$	$tg\alpha$
№62.	$\sin\left(\frac{3\pi}{2}-\alpha\right)tg(\alpha-\pi)-\cos\left(\frac{15\pi}{2}-\alpha\right)=-\cos\alpha\cdot tg\alpha-\cos\left(8\pi-\frac{\pi}{2}-\alpha\right)=$ $=-\sin\alpha-\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha+\sin\alpha=0$	0
№63.	$\cos\left(\frac{19\pi}{2}-\alpha\right)+\sin\left(\alpha-\frac{3\pi}{2}\right)tg(\pi-\alpha)=\cos\left(10\pi-\frac{\pi}{2}-\alpha\right)-\sin\left(\frac{3\pi}{2}-\alpha\right)\cdot tg(-\alpha)=$ $=\cos\left(-\left(\frac{\pi}{2}+\alpha\right)\right)+\cos\alpha\cdot(-tg\alpha)=\cos\left(\frac{\pi}{2}+\alpha\right)-\cos\alpha\cdot\frac{\sin\alpha}{\cos\alpha}=-\sin\alpha-\sin\alpha=$ $=-2\sin\alpha$	$-2\sin\alpha$

№64.	$$\frac{\cos(29{,}5\pi+2)\cdot ctg(19{,}5\pi-1)}{\sin\left(\sqrt{1-4\pi+4\pi^2}\right)\cos\left(\sqrt{16\pi^2+8\pi+1}\right)}=-2tg1$$ 1) $\cos\left(30\pi-\frac{\pi}{2}+2\right)=\cos\left(-\left(\frac{\pi}{2}-2\right)\right)=\cos\left(\frac{\pi}{2}-2\right)=\sin 2$ 2) $ctg\left(20\pi-\frac{\pi}{2}-1\right)=ctg\left(-\left(\frac{\pi}{2}+1\right)\right)=-ctg\left(\frac{\pi}{2}+1\right)=tg1$ 3) $\sin 2\cdot tg1=\frac{2\cdot\sin1\cdot\cos1\cdot\sin1}{\cos1}=2\cdot\sin1\cdot\sin1=2\cdot\sin^2 1$ 4) $\sin\left(\sqrt{(2\pi-1)^2}\right)=\sin\lvert 2\pi-1\rvert=\sin(2\pi-1)=\sin(-1)=-\sin1$ $т.к.\ 2\pi-1>0,\ то\ \lvert 2\pi-1\rvert=(2\pi-1)$ 5) $\cos\left(\sqrt{(4\pi+1)^2}\right)=\cos\lvert 4\pi+1\rvert=\cos1$ 6) $\frac{2\sin^2 1}{-\sin1\cdot\cos1}=-2tg1$	$-2tg1$
№65.	$$\frac{\sin(2-17{,}5\pi)\cdot tg(9{,}5\pi-1)}{\sin\left(\sqrt{4-4\pi+\pi^2}\right)\cos\left(\sqrt{4\pi^2+8\pi+4}\right)}=0{,}5\cdot\sin^{-2}1$$ 1) $\sin(-(-17{,}5\pi-2))=-\sin(18\pi-0{,}5\pi-2)=-\sin(-0{,}5\pi-2)=\sin\left(\frac{\pi}{2}+2\right)=\cos 2$ 2) $tg(10\pi-0{,}5\pi-1)=tg(-0{,}5\pi-1)=-tg\left(\frac{\pi}{2}+1\right)=ctg1$ 3) $\sin\sqrt{(\pi-2)^2}=\sin\lvert\pi-2\rvert=\sin(\pi-2)=\sin 2$ $\lvert\pi-2\rvert=\pi-2, т.к.\ \pi-2>0$ 4) $\cos\sqrt{(2\pi+2)^2}=\cos(2\pi+2)=\cos 2$ 5) $\frac{\cos2\cdot ctg1}{\sin2\cdot\cos2}=\frac{\cos1}{\sin1\cdot(2\cdot\sin1\cdot\cos1)}=0{,}5\cdot\sin^{-2}1$	$0{,}5\sin^{-2}1$
№66.	$$tg100^\circ+\frac{\sin530^\circ}{1+\sin640^\circ}$$ 1) $tg100^\circ=tg\left(90^\circ+10^\circ\right)=-ctg10^\circ$ 2) $\sin530^\circ=\sin\left(360^\circ+180^\circ-10^\circ\right)=\sin10^\circ$ 3) $\sin640^\circ=\sin\left(720^\circ-80^\circ\right)=-\sin80^\circ=-\cos10^\circ$ 4) $-ctg10^\circ+\frac{\sin10^\circ}{1-\cos10^\circ}=\frac{-\cos10^\circ}{\sin10^\circ}+\frac{\sin10^\circ}{1-\cos10^\circ}=$ $=\frac{-\cos10^\circ\left(1-\cos10^\circ\right)+\sin^2 10^\circ}{\sin10^\circ\cdot\left(1-\cos10^\circ\right)}=\frac{-\cos10^\circ+\cos^2 10^\circ+\sin^2 10^\circ}{\sin10^\circ\left(1-\cos10^\circ\right)}=$ $=\frac{1-\cos10^\circ}{\sin10^\circ\left(1-\cos10^\circ\right)}=\frac{1}{\sin10^\circ}$	$\frac{1}{\sin10^\circ}$

№67.	$ctg0,4\pi-\frac{\cos1,1\pi}{1-\cos0,6\pi}=\frac{1}{\cos0,1\pi}$ 1) $\cos1,1\pi=\cos(\pi+0,1\pi)=-\cos0,1\pi$ 2) $\cos0,6\pi=\cos(0,5\pi+0,1\pi)=-\sin0,1\pi$ 3) $ctg0,4\pi=\cos(0,5\pi-0,1\pi)=tg0,1\pi$ 4) $tg0,1\pi-\frac{-\cos0,1\pi}{1+\sin0,1\pi}=\frac{\sin0,1\pi}{\cos0,1\pi}+\frac{\cos0,1\pi}{1+\sin0,1\pi}=$ $=\frac{(1+\sin0,1\pi)\sin0,1\pi+\cos^2 0,1\pi}{\cos0,1\pi(1+\sin0,1\pi)}=$ $=\frac{\sin0,1\pi+\sin^2 0,1\pi+\cos^2 0,1\pi}{\cos0,1\pi(1+\sin0,1\pi)}=\frac{1+\sin0,1\pi}{\cos0,1\pi(1+\sin0,1\pi)}=\frac{1}{\cos0,1\pi}$	$\frac{1}{\cos0,1\pi}$
№68.	$tg(360^\circ-x)+ctg(270^\circ-x)+tg(180^\circ-x)+ctg(90^\circ-x)=$ $=tg(-x)+tgx-tgx+tgx=0$	0
№69.	$\sin\left(\frac{\pi}{2}+x\right)+\cos(\pi+x)+tg\left(\frac{3\pi}{2}-x\right)+ctg(2\pi-x)=$ $=\cos x-\cos x+ctgx-ctgx=0$	0
№70.	$\sin x-\sin(x-90^\circ)-\sin(x-180^\circ)-\sin(x-270^\circ)-\sin(x-360^\circ)=$ $=\sin x+\sin(90^\circ-x)+\sin(180^\circ-x)+\sin(270^\circ-x)-\sin x=$ $=\cos x-\sin x-\cos x=\sin x$	$\sin x$
№71.	$\cos(x+45^\circ)+\cos(x+135^\circ)+\cos(x+225^\circ)+\cos(x+315^\circ)$ *Пусть* $x+45^\circ=t$, *тогда* $\cos t+\cos(90^\circ+t)+\cos(t+180^\circ)+\cos(270^\circ+t)=$ $=\cos t-\sin t-\cos t+\sin t=0$	0
№72.	$tg(45^\circ-x)\cdot tg(45^\circ+x)$ *Т.к.* $(45^\circ-x)+(45^\circ+x)=90^\circ$, *то* $tg(45^\circ-x)=ctg(45^\circ+x)$ $ctg(45^\circ+x)\cdot tg(45^\circ+x)=1$	1
№73.	$ctg\left(\frac{\pi}{4}+x\right)\cdot ctg\left(\frac{\pi}{4}-x\right)$ *Т.к.* $\left(\frac{\pi}{4}+x\right)+\left(\frac{\pi}{4}-x\right)=\frac{\pi}{2}$, *то* $ctg\left(\frac{\pi}{4}+x\right)=tg\left(\frac{\pi}{4}-x\right)$ $tg\left(\frac{\pi}{4}-x\right)\cdot ctg\left(\frac{\pi}{4}-x\right)=1$	1

№74.	$\sin(90^\circ+x)\cdot\sin(180^\circ-x)\cdot\left(tg(180^\circ+x)+tg(270^\circ-x)\right)=$ $=\cos x\cdot\sin x\cdot(tgx+ctgx)=\cos x\cdot\sin x\cdot\left(\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}\right)=$ $=\frac{\sin x\cdot\cos x\left(\cos^2 x+\sin^2 x\right)}{\sin x\cdot\cos x}=1$	1
№75.	$\sin\left(x-\frac{\pi}{2}\right)\cdot\sin\left(x+\frac{\pi}{2}\right)-\sin^2(x-\pi)\cdot\sin^2(x+\pi)-\cos^2(x+\pi)\cdot\cos^2\left(x-\frac{3\pi}{2}\right)=$ $=-\sin\left(\frac{\pi}{2}-x\right)\cdot\cos x-\sin^2 x\cdot\sin^2 x-\cos^2 x\cdot\sin^2 x=$ $=-\cos^2 x-\sin^2 x\left(\sin^2 x+\cos^2 x\right)=-\left(\cos^2 x+\sin^2 x\right)=-1$	-1
№76.	$1-\sin(x-2\pi)\cdot\cos\left(x-\frac{3\pi}{2}\right)-tg(\pi-x)\cdot tg\left(\frac{3\pi}{2}-x\right)-2\cos^2(\pi+x)=$ $=1-\sin x\cdot\cos\left(\frac{3\pi}{2}-x\right)-tg(-x)\cdot ctgx-2\cos^2 x=$ $=1+\sin x\cdot\sin x+tgx\cdot ctgx-2\cos^2 x=1+\sin^2 x+1-2\cos^2 x=2-2\cos^2 x+\sin^2 x=$ $=2\left(1-\cos^2 x\right)+\sin^2 x=2\sin^2 x+\sin^2 x=3\sin^2 x$	$3\sin^2 x$
№77.	$\sin^2(\pi-x)+tg^2(\pi-x)\cdot tg^2\left(\frac{3\pi}{2}+x\right)+\sin\left(\frac{\pi}{2}+x\right)\cdot\cos(x-2\pi)=$ $=\sin^2 x+tg^2x\cdot ctg^2x+\cos x\cdot\cos x=\sin^2 x+1+\cos^2 x=2$	2
№78.	$\frac{ctg\left(\alpha-\frac{\pi}{2}\right)\left(\sin\left(\alpha-\frac{3\pi}{2}\right)-\sin(\pi+\alpha)\right)}{tg(\pi+\alpha)\left(\cos(\alpha+2\pi)+\sin(\alpha-2\pi)\right)}=$ $=\left(\frac{-ctg\left(\frac{\pi}{2}-\alpha\right)\cdot\left(-\sin\left(\frac{3\pi}{2}-\alpha\right)+\sin\alpha\right)}{tg\alpha\cdot(\cos\alpha+\sin\alpha)}\right)=\frac{-tg\alpha\cdot(\cos\alpha+\sin\alpha)}{tg\alpha\cdot(\cos\alpha+\sin\alpha)}=-1$	-1
№79.	$\frac{tg\left(\frac{3\pi}{2}-x\right)\cdot\cos\left(x-\frac{7\pi}{2}\right)}{\cos(10\pi-x)}+\cos(x-5\pi)\sin(5\pi-x)+\cos(5\pi+x)\sin\left(x-\frac{9\pi}{2}\right)=$ $=\frac{ctgx\cdot\cos\left(4\pi-\frac{\pi}{2}-x\right)}{\cos x}+\cos(\pi-x)\cdot\sin(\pi-x)+\cos(\pi+x)\cdot$ $\cdot\left(-\sin\left(4\pi+\frac{\pi}{2}-x\right)\right)=\frac{\cos x\cdot\cos\left(\frac{\pi}{2}+x\right)}{\sin x\cdot\cos x}-\cos x\cdot\sin x+\cos x\cdot\sin\left(\frac{\pi}{2}-x\right)=$ $=-\frac{\sin x}{\sin x}-\cos x\cdot\sin x+\cos x\cdot\cos x=$ $=-\cos x\cdot\sin x+\cos^2 x-1=-\cos x\cdot\sin x-\sin^2 x=-\sin x(\cos x+\sin x)$	$-\sin x\times$ $\times(\cos x+\sin x)$

№80.	$\left(ctg(6,5\pi-\alpha)\cdot\cos(-\alpha)+\cos(\pi-\alpha)\right)^2+\frac{2\sin^2(\pi-\alpha)}{tg(\alpha-\pi)}=$ $=\left(ctg\left(\frac{\pi}{2}-\alpha\right)\cdot\cos\alpha-\cos\alpha\right)^2+\frac{2\sin^2\alpha}{tg\alpha}=(tg\alpha\cdot\cos\alpha-\cos\alpha)^2+$ $+\frac{2\sin^2\alpha\cdot\cos\alpha}{\sin\alpha}=\left(\frac{\sin\alpha\cdot\cos\alpha}{\cos\alpha}-\cos\alpha\right)^2+2\cos\alpha\cdot\sin\alpha=$ $=(\sin\alpha-\cos\alpha)^2+2\cos\alpha\cdot\sin\alpha=\sin^2\alpha-2\sin\alpha\cdot\cos\alpha+$ $+\cos^2\alpha=2\cos\alpha\cdot\sin\alpha=1$	1
№81.	$\left(\sin\left(\frac{\pi}{2}+x\right)+\sin(\pi-x)\right)^2+\left(\cos(1,5\pi-x)+\cos(2\pi-x)\right)^2=$ $=(\cos x+\sin x)^2+(-\sin x+\cos x)^2=1+\sin 2x+1-\sin 2x=2$	2
№82.	$\left(\frac{\cos(2,5\pi+\alpha)}{ctg(3\pi-\alpha)}-\sin(-\alpha)\cdot tg\left(\frac{5\pi}{2}-\alpha\right)\right)^2+\frac{tg\alpha}{tg\left(\frac{3\pi}{2}+\alpha\right)}=$ $=\left(\frac{\cos\left(\frac{\pi}{2}+\alpha\right)}{ctg(-\alpha)}+\sin\alpha\cdot tg\left(\frac{\pi}{2}-\alpha\right)\right)^2+\frac{tg\alpha}{-ctg\alpha}=$ $=\left(\frac{-\sin\alpha}{-ctg\alpha}+\sin\alpha\cdot ctg\alpha\right)^2-\frac{\sin^2\alpha}{\cos^2\alpha}=\left(\frac{\sin\alpha\cdot\sin\alpha}{\cos\alpha}+\frac{\sin\alpha\cdot\cos\alpha}{\sin\alpha}\right)^2-\frac{\sin^2\alpha}{\cos^2\alpha}=$ $=\left(\frac{\sin^2\alpha+\cos^2\alpha}{\cos\alpha}\right)^2-\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha}-\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{1-\sin^2\alpha}{\cos^2\alpha}=\frac{\cos^2\alpha}{\cos^2\alpha}=1$	1
№83.	$\frac{tg\left(\frac{3\pi}{2}-\alpha\right)-\cos(\pi-\alpha)\sin(3\pi+\alpha)}{\left(\cos(3,5\pi+\alpha)+\sin(1,5\pi+\alpha)\right)^2-1}=\frac{ctg\alpha+\cos\alpha\cdot(-\sin\alpha)}{\left(\cos\left(\frac{\pi}{2}-\alpha\right)-\cos\alpha\right)^2-1}=$ $=\frac{ctg\alpha-\cos\alpha\cdot\sin\alpha}{(\sin\alpha-\cos\alpha)^2-1}=\frac{\frac{\cos\alpha}{\sin\alpha}-\cos\alpha\cdot\sin\alpha}{1-2\sin\alpha\cdot\cos\alpha-1}=\frac{\cos\alpha-\cos\alpha\cdot\sin^2\alpha}{-2\sin\alpha\cdot\cos\alpha\cdot\sin\alpha}=$ $=\frac{\cos\alpha\left(1-\sin^2\alpha\right)}{-2\sin^2\alpha\cdot\cos\alpha}=-\frac{\cos^2\alpha}{2\sin^2\alpha}=-\frac{1}{2}ctg^2\alpha$	$-\frac{1}{2}ctg^2\alpha$
№84.	$\frac{\sin 2\alpha}{\sin^2\left(\frac{\pi}{2}+\alpha\right)-\sin^2(\pi+\alpha)}$ и найти его числовое значение при $\alpha=\pi/8$. $\frac{\sin^2\alpha}{\cos^2\alpha-\sin^2\alpha}=\frac{\sin 2\alpha}{\cos 2\alpha}=tg2\alpha$ $\alpha=\frac{\pi}{8}$ $tg2\cdot\frac{\pi}{8}=tg\frac{\pi}{4}=1$	1

№85.	$\sqrt{2}\left(\sin^2\left(\frac{7\pi}{8}-2\alpha\right)-\sin^2\left(\frac{9\pi}{8}-2\alpha\right)\right)$ и найти его числовое значение при $\alpha=\pi/24$. $\boxed{\sin^2 x=\frac{1-\cos 2x}{2}}$ $\sqrt{2}\left(\sin^2\left(\pi-\frac{\pi}{8}-2\alpha\right)-\sin^2\left(\pi+\frac{\pi}{8}-2\alpha\right)\right)=$ $=\sqrt{2}\left(\sin^2\left(\pi-\left(\frac{\pi}{8}+2\alpha\right)\right)-\sin^2\left(\pi+\left(\frac{\pi}{8}-2\alpha\right)\right)\right)=$ $=\sqrt{2}\left(\sin^2\left(\frac{\pi}{8}+2\alpha\right)-\sin^2\left(\frac{\pi}{8}-2\alpha\right)\right)=$ $=\sqrt{2}\cdot\left(\frac{1-\cos\left(\frac{\pi}{4}+4\alpha\right)}{2}-\frac{1-\cos\left(\frac{\pi}{4}-4\alpha\right)}{2}\right)=$ $=\sqrt{2}\cdot\left(\frac{1}{2}-\frac{1}{2}-\frac{1}{2}\left(\cos\left(\frac{\pi}{4}+4\alpha\right)-\cos\left(\frac{\pi}{4}-4\alpha\right)\right)\right)=$ $=-\frac{\sqrt{2}}{2}\cdot 2\cdot\left(-\sin\frac{\frac{\pi}{4}+4\alpha-\frac{\pi}{4}+4\alpha}{2}\cdot\sin\frac{\frac{\pi}{2}}{2}\right)=$ $=\sqrt{2}\cdot\frac{\sqrt{2}}{2}\cdot\sin 4\alpha=\sin\frac{4\pi}{24}=\sin\frac{\pi}{6}=\frac{1}{2}$	0,5
№86.	$\frac{1}{tg^2\alpha}-\frac{2\cos 2\alpha}{1-\sin\left(2\alpha+\frac{\pi}{2}\right)}=\frac{1}{tg^2\alpha}-\frac{2\cos 2\alpha}{1-\cos 2\alpha}=\frac{\cos^2\alpha}{\sin^2\alpha}-\frac{2\left(\cos^2\alpha-\sin^2\alpha\right)}{2\sin^2\alpha}=$ $=\frac{\cos^2\alpha-\cos^2\alpha+\sin^2\alpha}{\sin^2\alpha}=1$	1
№87.	$\frac{1-2\cos^2\alpha}{2tg\left(\alpha-\frac{\pi}{4}\right)\cdot\sin^2\left(\frac{\pi}{4}+\alpha\right)}=\frac{1-2\cos^2\alpha}{2\cdot\left(-tg\left(\frac{\pi}{4}-\alpha\right)\right)\cdot\sin^2\left(\frac{\pi}{4}+\alpha\right)}=$ $=\frac{-\cos 2\alpha}{-2\cdot\frac{\sin\left(\frac{\pi}{4}-\alpha\right)}{\cos\left(\frac{\pi}{4}-\alpha\right)}\cdot\frac{\cos^2\left(\frac{\pi}{4}-\alpha\right)}{1}}=\frac{-\cos 2\alpha}{-\sin\left(\frac{\pi}{2}-2\alpha\right)}=*$ $Т.к.\ \left(\frac{\pi}{4}-\alpha\right)+\left(\frac{\pi}{4}+\alpha\right)=\frac{\pi}{2},\ то\ \ \sin^2\left(\frac{\pi}{4}+\alpha\right)=\cos^2\left(\frac{\pi}{4}-\alpha\right)$ $*=\frac{\cos 2\alpha}{\cos 2\alpha}=1$	1

III. Доказать тождества:

№88. $\cos(45^\circ + t) = \sin(45^\circ - t)$

$T.к.\ (45^\circ + t) + (45^\circ - t) = 90^\circ, то\ \cos(45^\circ + t) = \sin(45^\circ - t)$

$Или\ \cos(90^\circ - (45^\circ - t)) = \sin(45^\circ - t)$

$\cos(45^\circ - t) = \sin(45^\circ + t)$ аналогично

№90. $\sin(t-\pi)\cdot tg(t+\pi) + \dfrac{1}{\cos(t-2\pi)} = \cos t$

$-\sin(\pi - t)\cdot tgt + \dfrac{1}{\cos t} = \cos t$

$\dfrac{-\sin t\cdot \sin t}{\cos t} + \dfrac{1}{\cos t} = \cos t$

$\dfrac{1-\sin^2 t}{\cos t} = \cos t$

$\dfrac{\cos^2 t}{\cos t} = \cos t$

$\cos t = \cos t$

№91. $\sin(2\pi + t)\cdot ctg(3\pi + t) - \cos(2\pi - t)\cdot tg(3\pi - t) = \sin t + \cos t$

$\sin t\cdot ctgt - \cos t\cdot tg(-t) = \sin t + \cos t$

$\dfrac{\sin t\cdot \cos t}{\sin t} + \dfrac{\cos t\cdot \sin t}{\cos t} = \sin t + \cos t$

$\cos t + \sin t = \sin t + \cos t$

№92. $\sin 395^\circ \cdot \sin 505^\circ + \cos 575^\circ \cdot \cos 865^\circ + tg606^\circ \cdot tg1104^\circ = 2$

1) $\sin 395^\circ = \sin(360^\circ + 35^\circ) = \sin 35^\circ$

2) $\sin 505^\circ = \sin(360^\circ + 145^\circ) = \sin(180^\circ - 35^\circ) = \sin 35^\circ$

3) $\cos 575^\circ = \cos(360^\circ + 215^\circ) = \cos(180^\circ + 35^\circ) = -\cos 35^\circ$

4) $\cos 865^\circ = \cos(720^\circ + 145^\circ) = \cos(180^\circ - 35^\circ) = -\cos 35^\circ$

5) $tg606^\circ = tg(720^\circ - 114^\circ) = -tg(90^\circ + 24^\circ) = ctg24^\circ$

6) $tg1104^\circ = tg(360^\circ\cdot 3 + 24^\circ) = tg24^\circ$

7) $\sin 35^\circ \cdot \sin 35^\circ + (-\cos 35^\circ)\cdot(-\cos 35^\circ) + ctg24^\circ \cdot tg24^\circ = 2$

$\sin^2 35^\circ + \cos^2 35^\circ + 1 = 2$

$1 + 1 = 2$

$2 = 2$

№93.

$$\sin 405^\circ \cdot \cos 675^\circ + tg562^\circ \cdot tg788^\circ + \frac{1}{\cos 660^\circ} \cdot \frac{1}{\cos 1200^\circ} = -2,5$$

1) $\sin 405^\circ = \sin\left(360^\circ + 45^\circ\right) = \sin 45^\circ = \frac{\sqrt{2}}{2}$

2) $\cos 675^\circ = \cos\left(720^\circ - 45^\circ\right) = \cos 45^\circ = \frac{\sqrt{2}}{2}$

3) $tg562^\circ = tg\left(540^\circ + 22^\circ\right) = tg22^\circ$

4) $tg788^\circ = tg\left(720^\circ + 68^\circ\right) = tg68^\circ = ctg22^\circ$

5) $\cos 660^\circ = \cos\left(720^\circ - 60^\circ\right) = \cos 60^\circ = \frac{1}{2}$

6) $\cos 1200^\circ = \cos\left(360^\circ \cdot 3 + 120^\circ\right) = \cos 120^\circ = -\frac{1}{2}$

7) $\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} + tg22^\circ \cdot ctg22^\circ + \frac{1}{\frac{1}{2}} \cdot \frac{1}{\left(-\frac{1}{2}\right)} = \frac{1}{2} + 1 - 4 = 0,5 - 3 = -2,5$

№94.

$$\left(\sin\left(\frac{\pi}{2} - x\right) + \sin\left(\pi - x\right)\right)^2 + \left(\cos\left(\frac{3\pi}{2} - x\right) + \cos\left(2\pi - x\right)\right)^2 = 2$$

$$\left(\cos x + \sin x\right)^2 + \left(-\sin x + \cos x\right)^2 = 2$$

$$\sin^2 x + 2\sin x \cdot \cos x + \cos^2 x + \cos^2 x - 2\sin x \cdot \cos x + \sin^2 x = 2$$

$$1 + 1 = 2$$

$$2 = 2$$

№95.

$$\left(tg\frac{\pi}{4} + tg\left(\frac{\pi}{2} - x\right)\right)^2 + \left(ctg\frac{5\pi}{4} + ctg\left(\pi - x\right)\right)^2 = \frac{2}{\sin^2 x}$$

$$\left(1 + ctgx\right)^2 + \left(ctg\left(\pi + \frac{\pi}{4}\right) + ctg\left(-x\right)\right)^2 = \frac{2}{\sin^2 x}$$

$$1 + 2ctgx + ctg^2 x + \left(1 - ctgx\right)^2 = \frac{2}{\sin^2 x}$$

$$1 + 2ctgx + ctg^2 x + 1 - 2ctgx + ctg^2 x = \frac{2}{\sin^2 x}$$

$$2 + 2ctg^2 x = \frac{2}{\sin^2 x}$$

$$2\left(1 + ctg^2 x\right) = \frac{2}{\sin^2 x}$$

$$\frac{2}{\sin^2 x} = \frac{2}{\sin^2 x}$$

№96.

$$\sin\left(2\pi - x\right) \cdot tg\left(\frac{3\pi}{2} - x\right) - \cos\left(x - \pi\right) - \sin\left(x - \pi\right) = \sin x$$

$$\sin\left(-x\right) \cdot ctgx - \cos\left(\pi - x\right) + \sin\left(\pi - x\right) = \sin x$$

$$-\sin x \cdot \frac{\cos x}{\sin x} + \cos x + \sin x = \sin x$$

$$-\cos x + \cos x + \sin x = \sin x$$

$$\sin x = \sin x$$

№97.

$$\sin\left(\frac{\pi}{3}-t\right)\cdot tg\left(\frac{2\pi}{3}+t\right)\cdot\cos\left(\frac{5\pi}{3}+t\right)+tg\left(\pi+t\right)\cdot tg\left(\frac{3\pi}{2}-t\right)=\cos^2\left(\frac{\pi}{3}-t\right)$$

$$y=\frac{\pi}{3}-t$$

1) $tg\left(\pi-\frac{\pi}{3}+t\right)=tg\left(\pi-\left(\frac{\pi}{3}-t\right)\right)=tg\left(\pi-y\right)=-tgy$

2) $\cos\left(\frac{5\pi}{3}+t\right)=\cos\left(2\pi-\frac{\pi}{3}+t\right)=\cos\left(2\pi-\left(\frac{\pi}{3}-t\right)\right)=\cos y$

3) $tg\left(\pi+t\right)=tgt$

4) $tg\left(\frac{3\pi}{2}-t\right)=ctgt$

5) $\sin y\cdot\left(-tgy\right)\cdot\cos y+tgy\cdot ctgy=\cos^2 y$

$$\frac{-\sin y\cdot\sin y}{\cos y}\cdot\cos y+1=\cos^2 y$$

$$1-\sin^2 y=\cos^2 y$$

$$1=1$$

№98.

$$\frac{\sin\left(x-\pi\right)\cdot\cos\left(x-2\pi\right)\cdot\sin\left(2\pi-x\right)}{\sin\left(\frac{\pi}{2}-x\right)\cdot ctg\left(\pi-x\right)ctg\left(\frac{3\pi}{2}+x\right)}=\sin^2 x$$

$$\frac{-\sin\left(\pi-x\right)\cdot\cos x\cdot\sin\left(-x\right)}{\cos x\cdot ctg\left(-x\right)\cdot\left(-tgx\right)}=\sin^2 x$$.

$$\frac{\sin x\cdot\sin x}{1}=\sin^2 x$$

$$\sin^2 x=\sin^2 x$$

№99.

$$\frac{\sin\left(\pi+x\right)\cdot\cos\left(\frac{3\pi}{2}-x\right)\cdot tg\left(x-\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}+x\right)\cdot tg\left(\pi+x\right)\cos\left(\frac{3\pi}{2}+x\right)}=ctg^2x$$

$$\frac{-\sin x\cdot\left(-\sin x\right)\cdot\left(-tg\left(\frac{\pi}{2}-x\right)\right)}{-\sin x\cdot tgx\cdot\sin x}=ctg^2x$$

$$ctgx\cdot ctgx=ctg^2x$$

$$ctg^2x=ctg^2x$$

IV. Вычислить значение выражения при условии:

№100.	Вычислить $\sin\left(\frac{\pi}{2}+\alpha\right)$, если $\sin(\pi+\alpha)=0{,}8$ и $\alpha\in\left(-\frac{\pi}{2};0\right)$. $\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha=?$ $\sin(\pi+\alpha)=-\sin\alpha=0{,}8$ $\sin\alpha=-0{,}8$ $\alpha\in\left(-\frac{\pi}{2};0\right),\quad \alpha\in IV\Rightarrow\cos\alpha>0$ $\left.\begin{array}{l}\cos\alpha=\pm\sqrt{1-\sin^2\alpha}\\ \cos\alpha>0\end{array}\right\vert\Rightarrow\cos\alpha=\sqrt{1-0{,}8^2}=0{,}6$	0,6
№101.	Вычислить $\cos\left(\frac{3\pi}{2}+2\alpha\right)$, если $\sin\alpha-\cos\alpha=0{,}5$. $\cos\left(\frac{3\pi}{2}+2\alpha\right)=\sin2\alpha=?$ $\sin\alpha-\cos\alpha=0{,}5\quad\uparrow^2$ $1-\sin2\alpha=0{,}25$ $\sin2\alpha=0{,}75$	0,75
№102.	Вычислить $\sin2x$, если $\sin x+\sin(2{,}5\pi+x)=0{,}2$. $\sin2x=?$ $\sin x+\sin\left(2\pi+\frac{\pi}{2}+x\right)=0{,}2$ $\sin x+\sin\left(\frac{\pi}{2}+x\right)=0{,}2$ $\sin x+\cos x=0{,}2\quad\uparrow^2$ $1+\sin2x=0{,}04$ $\sin2x=0{,}04-1$ $\sin2x=-0{,}96$	-0,96
№103.	Вычислить $\sin2x$, если $\sin x+\sin(3{,}5\pi+x)=0{,}2$. $\sin2x=?$ $\sin x+\sin\left(2\pi+\frac{3\pi}{2}+x\right)=0{,}2$ $\sin x-\cos x=0{,}2\quad\uparrow^2$ $1-\sin2x=0{,}04$ $\sin2x=0{,}96$	0,96

№104.	Вычислить tg^2x+ctg^2x, если $tgx+tg\left(\frac{3\pi}{2}-x\right)=5$. $tg^2x+ctg^2x=?$ $tgx+ctgx=5 \quad \uparrow^2$ $tg^2x+2\cdot tgx\cdot ctgx+ctg^2x=25$ $tg^2x+ctg^2x=25-2$ $tg^2x+ctg^2x=23$	23
№105.	Вычислить tg^2x+ctg^2x, если $tg(\pi-x)+ctgx=5$. $tg^2x+ctg^2x=?$ $tg(-x)+ctgx=5$ $ctgx-tgx=5 \quad \uparrow^2$ $ctg^2x-2\cdot tgx\cdot ctgx+tg^2x=25$ $ctg^2x+tg^2x=27$	27
№106.	Вычислить $\left(tg\left(\frac{5\pi}{4}+x\right)+tg\left(\frac{5\pi}{4}-x\right)\right)^{-1}$, если $tg\left(\frac{3\pi}{2}+x\right)=\frac{3}{4}$. 1) $tg\left(\frac{5\pi}{4}+x\right)=tg\left(\pi+\frac{\pi}{4}+x\right)=tg\left(\frac{\pi}{4}+x\right)$ 2) $tg\left(\frac{5\pi}{4}-x\right)=tg\left(\pi+\frac{\pi}{4}-x\right)=tg\left(\frac{\pi}{4}-x\right)$ 3) $Т.к.\ \left(\frac{\pi}{4}+x\right)+\left(\frac{\pi}{4}-x\right)=\frac{\pi}{2},\ то\ tg\left(\frac{\pi}{4}-x\right)=ctg\left(\frac{\pi}{4}+x\right)$ 4) $\left(tg\left(\frac{\pi}{4}+x\right)=ctg\left(\frac{\pi}{4}+x\right)\right)^{-1}=\left(\frac{\sin\left(\frac{\pi}{4}+x\right)}{\cos\left(\frac{\pi}{4}+x\right)}+\frac{\cos\left(\frac{\pi}{4}+x\right)}{\sin\left(\frac{\pi}{4}+x\right)}\right)^{-1}=$ $=\left(\frac{\sin^2\left(\frac{\pi}{4}+x\right)+\cos^2\left(\frac{\pi}{4}+x\right)}{\sin\left(\frac{\pi}{4}+x\right)\cdot\cos\left(\frac{\pi}{4}+x\right)}\right)^{-1}=\frac{2\cdot\sin\left(\frac{\pi}{4}+x\right)\cdot\cos\left(\frac{\pi}{4}+x\right)}{2}=$ $=\frac{\sin 2\cdot\left(\frac{\pi}{4}+x\right)}{2}=\frac{\sin\left(\frac{\pi}{2}+2x\right)}{2}=\frac{\cos 2x}{2}$ 5) $tg\left(\frac{3\pi}{2}+x\right)=-ctgx,\quad ctgx=-\frac{3}{4};\ tgx=-\frac{4}{3}$ 6) $\cos 2x=\frac{1-tg^2x}{1+tg^2x}=\frac{1-\left(-\frac{4}{3}\right)^2}{1+\frac{16}{9}}=\left(1-\frac{16}{9}\right):\frac{25}{9}=-\frac{7}{9}\cdot\frac{9}{25}=-\frac{7\cdot 4}{25\cdot 4}=-0,28$ 7) $-\frac{0,28}{2}=-0,14$	-0,14

V. Разные задачи.

№107.	При каком значении *a* число $\frac{\pi}{4}$ является корнем уравнения $\sin^2 x + a\cdot\sin x\cdot\cos x - 3\cos^2 x = 0,\quad x=\frac{\pi}{4}$ $\sin^2\frac{\pi}{4} + a\cdot\sin\frac{\pi}{4}\cdot\cos\frac{\pi}{4} - 3\cos^2\frac{\pi}{4} = 0$ $\left(\frac{\sqrt{2}}{2}\right)^2 + a\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} - 3\cdot\left(\frac{\sqrt{2}}{2}\right)^2 = 0$ $a\cdot\frac{2}{4} = 3\cdot\frac{2}{4} - \frac{2}{4}$ $\frac{a}{2} = \frac{3}{2} - \frac{1}{2}$ $a = 2$	2
№108.	При каком значении *a* число $\frac{3\pi}{4}$ является корнем уравнения $\sin^2 x + a\cdot\sin x\cdot\cos x + 2\cos^2 x = 0,\quad x=\frac{3\pi}{4}$ $\sin^2\left(\frac{3\pi}{4}\right) + a\cdot\sin\frac{3\pi}{4}\cdot\cos\frac{3\pi}{4} + 2\cos^2\frac{3\pi}{4} = 0$ $\left(\frac{\sqrt{2}}{2}\right)^2 + a\cdot\left(\frac{\sqrt{2}}{2}\right)\cdot\left(-\frac{\sqrt{2}}{2}\right) + 2\cdot\left(\frac{\sqrt{2}}{2}\right)^2 = 0$ $\frac{1}{2} - a\cdot\frac{1}{2} + 1 = 0$ $-\frac{a}{2} = -1 - \frac{1}{2}$ $a = 3$	3
№109.	При каком значении а выражение $\sin^2 x - \cos\left(\frac{\pi}{2} - x\right)\sin\left(\frac{a\pi}{4} - x\right)$ обращается в нуль при любом значении x. $\sin^2 x - \cos\left(\frac{\pi}{2} - x\right)\sin\left(\frac{a\pi}{4} - x\right) = 0$ $\sin^2 x = \sin x\cdot\sin\left(\frac{a\pi}{4} - x\right)$ *Равенство будет верным при* $\forall x$, *если* $\sin\left(\frac{a\pi}{4} - x\right) = \sin x$ $\frac{a\pi}{4} = \pi$ $a = 4$ $\sin(\pi - x) = \sin x$ $\sin x = \sin x$	4

№110.	При каком значении а выражение $\cos^2 x - \sin\left(\frac{\pi}{2} - x\right)\sin\left(\frac{a\pi}{4} - x\right)$ обращается в нуль при любом значении х. $\cos^2 x - \sin\left(\frac{\pi}{2} - x\right)\sin\left(\frac{a\pi}{4} - x\right) = 0$ $\cos^2 x - \cos x \cdot \sin\left(\frac{a\pi}{4} - x\right) = 0$ $\sin\left(\frac{a\pi}{4} - x\right) = \cos x$ $\sin\left(\frac{\pi}{2} - x\right) = \cos x, \quad \frac{a\pi}{4} = \frac{\pi}{2}$ $a = 2$	2
№111.	Построить график функции $y = \sqrt{1 + tg^2 x} \cdot \frac{\cos^2(-x) \cdot \cos\left(\frac{\pi}{2} - x\right)}{tg\left(\frac{\pi}{2} - x\right) \cdot \sin^2(\pi + x)}$ $y = \sqrt{\frac{1}{\cos^2 x}} \cdot \frac{\cos^2 x \cdot \sin x}{ctgx \cdot \sin^2 x}$ $ОДЗ: \begin{cases} \cos x \neq 0 \\ \sin x \neq 0 \end{cases}$ $x \neq \frac{\pi k}{2}$ $y = \frac{1}{\lvert\cos x\rvert} \cdot \frac{\cos^2 x \cdot \sin x}{\cos x \cdot \sin x}$ $y = \frac{\cos x}{\lvert\cos x\rvert} \Leftrightarrow \left[\begin{array}{l} \cos x > 0, \quad y = 1 \\ \cos x < 0, \quad y = -1 \end{array}\right.$	$y = \frac{\cos x}{\lvert\cos x\rvert}$ $x \neq \frac{\pi n}{2}$
№112.	Построить график функции $y = \sqrt{1 + ctg^2 x} \cdot \frac{ctg\left(\frac{\pi}{2} - x\right) \cdot \cos\left(\frac{3\pi}{2} + x\right)}{tg(\pi - x)}$ $y = \sqrt{\frac{1}{\sin^2 x}} \cdot \frac{tgx \cdot \sin x}{-tgx}$ $ОДЗ: \quad x \neq \frac{\pi k}{2}$ $y = \frac{-\sin x}{\lvert\sin x\rvert} \Leftrightarrow \left[\begin{array}{l} \sin x > 0, \quad y = -1 \\ \sin x < 0, \quad y = 1 \end{array}\right.$	$y = -\frac{\sin x}{\lvert\sin x\rvert}$ $x \neq \frac{\pi k}{2}$

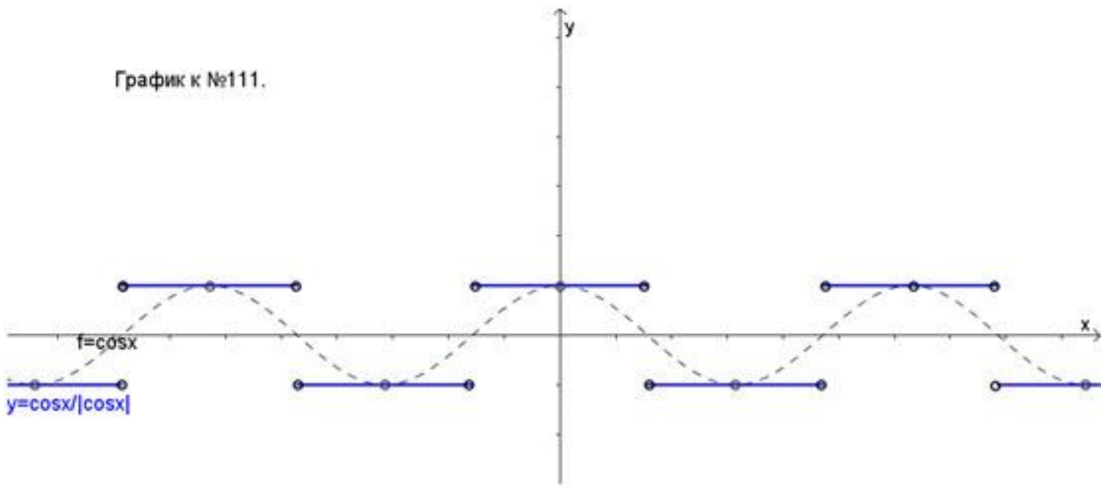
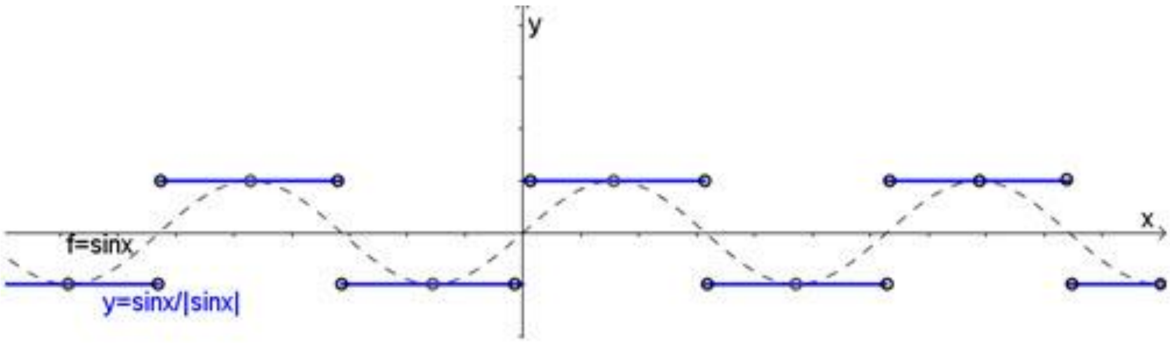
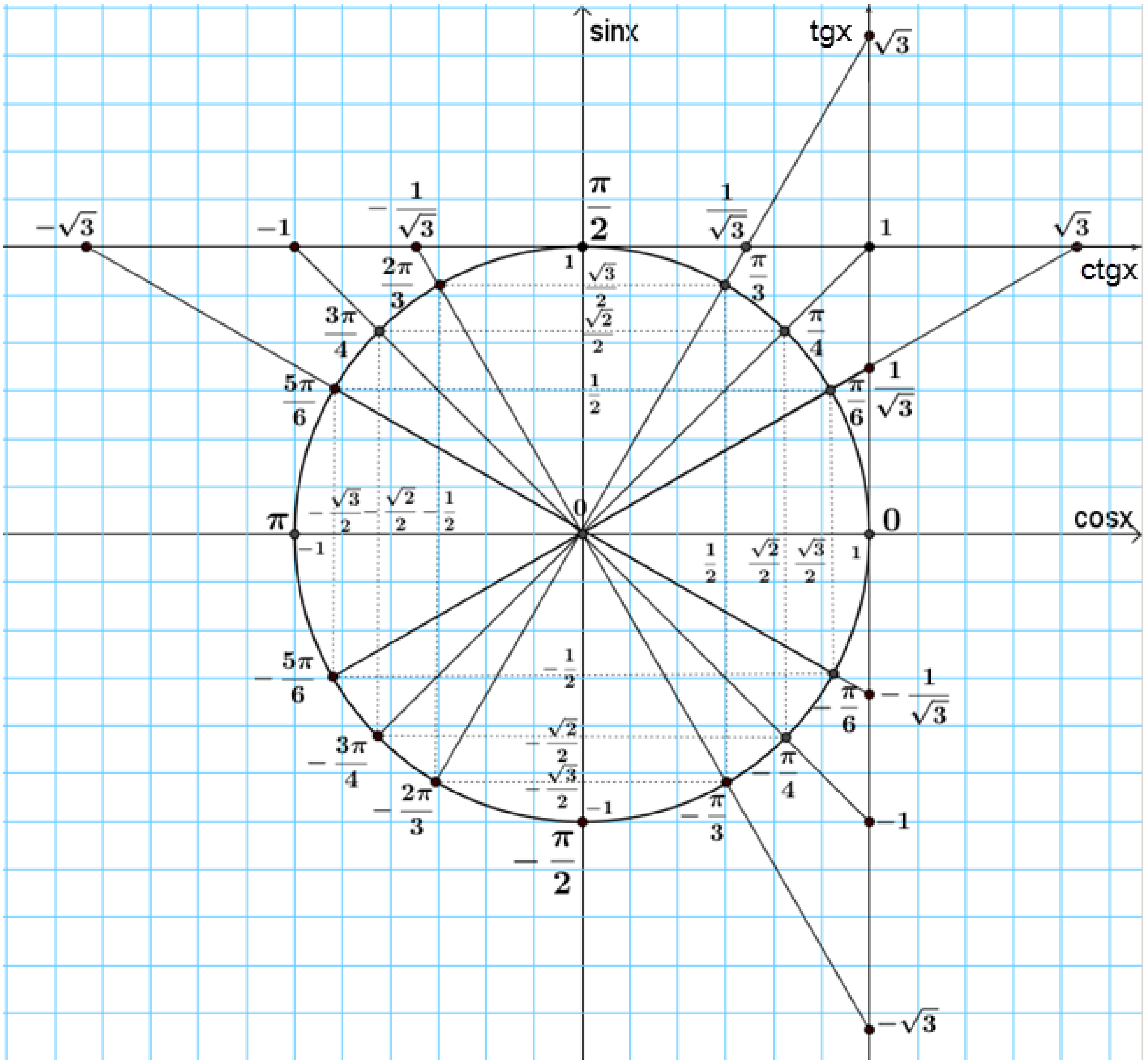


График к №112.



✓ *Тригонометрическая окружность*



✓ Основные тригонометрические формулы

1. $\sin^2\alpha+\cos^2\alpha=1$ **2.** $tg^2\alpha+1=\dfrac{1}{\cos^2\alpha}$ **3.** $ctg^2\alpha+1=\dfrac{1}{\sin^2\alpha}$ **4.** $tg\alpha\cdot ctg\alpha=1$

5. $\sin(\alpha-\beta)=\sin\alpha\cdot\cos\beta-\sin\beta\cdot\cos\alpha$ **6.** $\sin(\alpha+\beta)=\sin\alpha\cdot\cos\beta+\sin\beta\cdot\cos\alpha$

7. $\cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta$ **8.** $\cos(\alpha+\beta)=\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta$

9. $tg(\alpha-\beta)=\dfrac{tg\alpha-tg\beta}{1+tg\alpha\cdot tg\beta}$ **10.** $tg(\alpha+\beta)=\dfrac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta}$

11. $\sin2\alpha=2\sin\alpha\cdot\cos\alpha$ **12.** $tg2\alpha=\dfrac{2tg\alpha}{1-tg^2\alpha}$

13. $\cos2\alpha=\cos^2\alpha-\sin^2\alpha=2\cos^2\alpha-1=1-2\sin^2\alpha$

14. $1+\cos2\alpha=2\cos^2\alpha$ **15.** $1-\cos2\alpha=2\sin^2\alpha$

16. $\cos^2\alpha=\dfrac{1+\cos2\alpha}{2}$ **17.** $\sin^2\alpha=\dfrac{1-\cos2\alpha}{2}$

18. $\sin\alpha-\sin\beta=2\sin\dfrac{\alpha-\beta}{2}\cos\dfrac{\alpha+\beta}{2}$ **19.** $\sin\alpha+\sin\beta=2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

20. $\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$ **21.** $\cos\alpha-\cos\beta=-2\sin\dfrac{\alpha-\beta}{2}\sin\dfrac{\alpha+\beta}{2}$

22. $\sin\alpha\cdot\cos\beta=\dfrac{1}{2}\left(\sin(\alpha-\beta)+\sin(\alpha+\beta)\right)$ **23.** $\sin\alpha\cdot\sin\beta=\dfrac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)$

24. $\cos\alpha\cdot\cos\beta=\dfrac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)$

25. $\left|\cos\dfrac{\alpha}{2}\right|=\sqrt{\dfrac{1+\cos\alpha}{2}}$ **26.** $\left|\sin\dfrac{\alpha}{2}\right|=\sqrt{\dfrac{1-\cos\alpha}{2}}$ **27.** $\left|tg\dfrac{\alpha}{2}\right|=\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

28. $\sin2\alpha=\dfrac{2tg\alpha}{1+tg^2\alpha}$ **29.** $\cos2\alpha=\dfrac{1-tg^2\alpha}{1+tg^2\alpha}$

30. $a\cdot\cos x+b\cdot\sin x=\sqrt{a^2+b^2}\sin(x+\varphi),\ \sin\varphi=\dfrac{a}{\sqrt{a^2+b^2}},\cos\varphi=\dfrac{b}{\sqrt{a^2+b^2}}$

✓ *Алгоритм применения формул приведения*

1. *Исследуем функцию на четность/нечетность.*

$$\cos(-t) = \cos t$$
$$\sin(-t) = -\sin t$$
$$tg(-t) = -tgt$$
$$ctg(-t) = -ctgt$$

2. *Исследуем функцию на периодичность.*

$$T_{\cos t} = T_{\sin t} = 2\pi \qquad T_{tgt} = T_{ctgt} = \pi$$

или

$$\sin(t + 2\pi k) = \sin t$$
$$\cos(t + 2\pi k) = \cos t$$
$$tg(t + \pi k) = tgt$$
$$ctg(t + \pi k) = ctgt$$
$$k \in \mathbb{Z}$$

3. *Представим угол в виде:* $\left(\frac{\pi}{2} \pm t\right)$, $\left(\frac{3\pi}{2} \pm t\right)$, $(\pi \pm t)$ *или* $(2\pi \pm t)$, *где* $t \in I$.

4. ***Определим знак исходной функции и поставим его перед приводимой функцией.***
5. ***Для углов вида***
   - $\left(\frac{\pi}{2} \pm t\right)$ ***или*** $\left(\frac{3\pi}{2} \pm t\right)$ ***название функции изменяем на “ко-функцию”;***

   для углов вида
   - $(\pi \pm t)$ ***или*** $(2\pi \pm t)$ ***название функции*** **не изменяем.**