Тренировочные задания

Алгебра 10

Преобразование тригонометрических выражений

№	Докажите тождества	
1.	а) $(1+tgx)^2-2tgx=\frac{1}{\cos^2 x}$ $1+2tgx+tg^2x-2tgx=\frac{1}{\cos^2 x}$ $1+tg^2x=\frac{1}{\cos^2 x}$ $\frac{1}{\cos^2 x}=\frac{1}{\cos^2 x}$	б) $(1+ctgx)^2-\frac{1}{\sin^2 x}=2ctgx$ $1+2ctgx+ctg^2x-\frac{1}{\sin^2 x}=2ctgx$ $\frac{1}{\sin^2 x}+2ctgx-\frac{1}{\sin^2 x}=2ctgx$ $2ctgx=2ctgx$
2.	а) $\frac{tgx+1}{\sin x+\cos x}=\frac{1}{\cos x}$ $\frac{\frac{\sin x}{\cos x}+1}{\sin x+\cos x}=\frac{1}{\cos x}$ $\frac{\sin x+\cos x}{\cos x}:(\sin x+\cos x)=\frac{1}{\cos x}$ $\frac{\sin x+\cos x}{\cos x}\cdot\frac{1}{\sin x+\cos x}=\frac{1}{\cos x}$ $\frac{1}{\cos x}=\frac{1}{\cos x}$	б) $\frac{ctgx-1}{\cos x-\sin x}=\frac{1}{\sin x}$ $\frac{\frac{\cos x}{\sin x}-1}{\cos x-\sin x}=\frac{1}{\sin x}$ $\frac{\cos x-\sin x}{\sin x}:(\cos x-\sin x)=\frac{1}{\sin x}$ $\frac{\cos x-\sin x}{\sin x}\cdot\frac{1}{\cos x-\sin x}=\frac{1}{\sin x}$ $\frac{1}{\sin x}=\frac{1}{\sin x}$
3.	а) $\frac{\cos^4 x+\sin^2 x\cdot\cos^2 x}{\sin^2 x}=ctg^2x$ $\frac{\cos^2 x(\cos^2 x+\sin^2 x)}{\sin^2 x}=ctg^2x$ $\frac{\cos^2 x}{\sin^2 x}=ctg^2x$ $ctg^2x=ctg^2x$	б) $\frac{\sin^4 x+\sin^2 x\cdot\cos^2 x}{\cos^2 x}=tg^2x$ $\frac{\sin^2 x(\sin^2 x+\cos^2 x)}{\cos^2 x}=tg^2x$ $\frac{\sin^2 x}{\cos^2 x}=tg^2x$ $tg^2x=tg^2x$

4.	а) $\frac{1+tgx}{1+ctgx}=tgx$	б) $\frac{1-ctgx}{1-tgx}=-ctgx$
	$\left(1+\frac{\cos x}{\sin x}\right):\left(1+\frac{\sin x}{\cos x}\right)=tgx$ $\frac{\cos x+\sin x}{\cos x}:\frac{\sin x+\cos x}{\sin x}=tgx$ $\frac{\cos x+\sin x}{\cos x}\cdot\frac{\sin x}{\sin x+\cos x}=tgx$ $\frac{\sin x}{\cos x}=tgx,\quad tgx=tgx$	$\frac{1-\frac{\cos x}{\sin x}}{1-\frac{\sin x}{\cos x}}=-ctgx$ $\frac{\sin x-\cos x}{\sin x}:\frac{\cos x-\sin x}{\cos x}=-ctgx$ $\frac{\sin x-\cos x}{\sin x}\cdot\frac{\cos x}{-(\sin x-\cos x)}=-ctgx$ $-\frac{\cos x}{\sin x}=-ctgx,\quad -ctgx=-ctgx$
5.	а) $\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}=\frac{2}{\sin x}$ $\frac{\sin^2 x+(1+\cos x)^2}{(1+\cos x)\sin x}=\frac{2}{\sin x}$ $\frac{\sin^2 x+1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}=\frac{2}{\sin x}$ $\frac{2+2\cos x}{(1+\cos x)\sin x}=\frac{2}{\sin x}$ $\frac{2(1+\cos x)}{(1+\cos x)\sin x}=\frac{2}{\sin x}$ $\frac{2}{\sin x}=\frac{2}{\sin x}$	б) $\frac{\sin x}{1+\cos x}+\frac{\sin x}{1-\cos x}=\frac{2}{\sin x}$ $\frac{\sin x(1-\cos x)+\sin x(1+\cos x)}{(1+\cos x)(1-\cos x)}=\frac{2}{\sin x}$ $\frac{\sin x-\sin x\cdot\cos x+\sin x+\sin x\cdot\cos x}{1-\cos^2 x}=\frac{2}{\sin x}$ $\frac{2\sin x}{\sin^2 x}=\frac{2}{\sin x}$ $\frac{2}{\sin x}=\frac{2}{\sin x}$
	Упростите выражения	
6.	а) $\frac{\cos x-\cos^3 x}{\sin^2 x}$ $\frac{\cos x(1-\cos^2 x)}{\sin^2 x}=\frac{\cos x\cdot\sin^2 x}{\sin^2 x}=\cos x$	б) $\frac{\sin x-\sin^3 x}{\cos^2 x}$ $\frac{\sin x(1-\sin^2 x)}{\cos^2 x}=\frac{\sin x\cdot\cos^2 x}{\cos^2 x}=\sin x$
7.	а) $\frac{(1-\cos x)(1+\cos x)}{\sin x}$ $\frac{1-\cos^2 x}{\sin x}=\frac{\sin^2 x}{\sin x}=\sin x$	б) $\frac{(1-\sin x)(1+\sin x)}{\cos x}$ $\frac{1-\sin^2 x}{\cos x}=\frac{\cos^2 x}{\cos x}=\cos x$
8.	а) $\frac{1}{\cos^2 x}-tgx\cdot ctgx$ $\frac{1}{\cos^2 x}-1=\frac{1-\cos^2 x}{\cos^2 x}=\frac{\sin^2 x}{\cos^2 x}=tg^2x$	б) $tgx\cdot ctgx-\frac{1}{\sin^2 x}$ $1-\frac{1}{\sin^2 x}=\frac{\sin^2 x-1}{\sin^2 x}=\frac{-\cos^2 x}{\sin^2 x}=-ctg^2x$
9.	а) $tg^2+\sin^2 x-\frac{1}{\cos^2 x}$ $tg^2x+\sin^2 x-(1+tg^2x)=$ $=tg^2x+\sin^2 x-1-tg^2x=-\cos^2 x$	б) $ctg^2+\cos^2 x-\frac{1}{\sin^2 x}$ $ctg^2x+\cos^2 x-(1+ctg^2x)=$ $=ctg^2x+\cos^2 x-1-ctg^2x=-\sin^2 x$

10.	а) $(1+tgx)^2+(1-tgx)^2$ $1+2tgx+tg^2x+1-2tgx+tg^2x=$ $=2+2tg^2x=2(1+tg^2x)=\dfrac{2}{\cos^2 x}$	б) $(1+ctgx)^2+(1-ctgx)^2$ $1-2ctgx+ctg^2x+1+2ctgx+ctg^2x=$ $=2+2ctg^2x=2(1+ctg^2x)=\dfrac{2}{\sin^2 x}$
11.	а) $\sin^4 x+\sin^2 x\cdot\cos^2 x-\sin^2 x+1$ $\sin^2 x(\sin^2 x+\cos^2 x)+1-\sin^2 x=$ $=\sin^2 x+\cos^2 x=1$	б) $\sin^4 x-\cos^4 x-\sin^2 x+\cos^2 x$ $(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)-\sin^2 x+\cos^2 x=$ $=\sin^2 x-\cos^2 x-\sin^2 x+\cos^2 x=0$
12.	а) $\left(\dfrac{\sin x}{tgx}\right)^2+\left(\dfrac{\cos x}{ctgx}\right)^2-\sin^2 x$ $\left(\sin x:\dfrac{\sin x}{\cos x}\right)^2+\left(\cos x:\dfrac{\cos x}{\sin x}\right)^2-\sin^2 x=$ $=\cos^2 x+\sin^2 x-\sin^2 x=\cos^2 x$	б) $(tgx\cos x)^2+(ctgx\sin x)^2-\cos^2 x$ $\left(\dfrac{\sin x}{\cos x}\cdot\cos x\right)^2+\left(\dfrac{\cos x}{\sin x}\cdot\sin x\right)^2-\cos^2 x=$ $=\cos^2 x+\sin^2 x-\cos^2 x=\sin^2 x$
13.	а) $\dfrac{\cos x}{1+\sin x}+tgx$ $\dfrac{\cos x}{1+\sin x}+\dfrac{\sin x}{\cos x}=\dfrac{\cos^2 x+\sin x+\sin^2 x}{(1+\sin x)\cdot\cos x}=$ $=\dfrac{1+\sin x}{(1+\sin x)\cdot\cos x}=\dfrac{1}{\cos x}$	б) $ctgx+\dfrac{\sin x}{1+\cos x}$ $\dfrac{\cos x}{\sin x}+\dfrac{\sin x}{1+\cos x}=\dfrac{\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)}=$ $=\dfrac{1+\cos x}{\sin x(1+\cos x)}=\dfrac{1}{\sin x}$
14.	а) $\dfrac{1-\sin^2 x}{1-\cos^2 x}+tgx\cdot ctgx$ $\dfrac{\cos^2 x}{\sin^2 x}+1=\dfrac{\cos^2 x+\sin^2 x}{\sin^2 x}=\dfrac{1}{\sin^2 x}$	б) $(1-\cos^2 x)tg^2x+1-tg^2x$ $tg^2x-\cos^2 x\cdot\dfrac{\sin^2 x}{\cos^2 x}+1-tg^2x=1-\sin^2 x=\cos^2 x$